MATH 126 C/D MIDTERM 2 Spring 2016

May 17, 2016

NAME: SIGNATURE:

STUDENT ID #: TA SECTION:

Problem	Number of points	Points obtained
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Instructions:

- Your exam consists of FIVE problems. Please check that you have all of them.
- No books or notebooks allowed; you may use an A4 double-sided, handwritten sheet of notes *for personal use* (do not share).
- Place a box around your final answer to each question.
- The only calculator allowed is the **TI 30X IIS** of any color. All other electronic devices are prohibited.
- **Answers with little or no justification may receive no credit.**
- **Answers obtained by guess-and-check work will receive little or no credit, even if correct.**
- Read problems *carefully.*
- Raise your hand if you have a question.
- If you need more space, use additional blank sheets which will be provided by your TA. It is your responsibility to have him/her staple the additional sheets to your exam before you turn it in.
- Please turn off cell phones. GOOD LUCK!

Problem 1. (10 pts) Consider the polar curve

$$r + \frac{1}{4r} = \sin(\theta) + \cos(\theta).$$

Show by writing the Cartesian equation that this curve is a circle. What is the center and the radius of this circle?

Problem 2. (4+3+3=10 pts) (curvature) Answer the following multiple choice questions. You need not show any work.

(a) (4 pts) Suppose $\mathbf{r}(t)$ is a vector valued function in $\mathbb{R}^3$. Which among the statements must be true? Give all correct responses.

(i) If the curvature $\kappa(t) = 0$ for some t, then the normal component of acceleration must be zero.

(ii) The velocity $\mathbf{r}'(t)$ must be perpendicular to the unit normal vector $N(t)$.

(iii) If $\mathbf{T}(t)$ is the unit tangent vector, we must have $|\mathbf{T}'(t)| = 1$.

(iv) If the curvature takes the same value at every point, we must be traveling along a straight line or an arc of a circle.

(b) (3 pts) Now suppose we go along the same path as $\mathbf{r}$ above but travel at exactly half the speed as before? There is exactly one statement below which is true. Identify it.

(i) The new curvature is half of the old curvature at every point.

(ii) The new curvature is twice the old curvature at every point.

(iii) The new curvature is exactly the same as the old curvature.

(iv) None of the relations above hold in general. It depends on the curve.

(c) (3 pts) Suppose the binormal vector is $\langle 0, 0, 1 \rangle$. Which of the following statements must be true? Give all correct answers.

(i) The osculating plane at every point is parallel to the xy-plane.

(ii) The z-component of the velocity $\mathbf{r}'(t)$ must be zero at every t.

(iii) We must be traveling along a straight line or an arc of a circle.

Problem 3. (5+3+2=10 pts) Find an equation of the tangent plane to the surface $z = \ln(x - 9y)$ at the point $(10, 1, 0)$. Use differentials to approximate the value of z when $x = 10.5, y = 1$. Explain why the same method does not work to approximate the value of z when $x = 10, y = 1.5$.

Problem 4. (10 pts) Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(2, 2, 0)$.

Problem 5. (10 pts) Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = [0, 1] \times [-2, 3]$.

Extra sheet.