Solutions to Math 126A MT2 Sp18

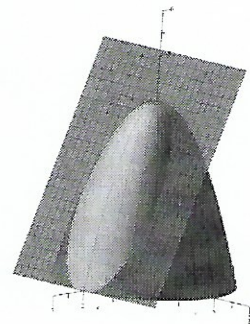
1. (7 points) Set up an integral in **polar** coordinates to find the volume of the solid under the paraboloid

$$z = 4 - x^2 - y^2$$

and above the plane

$$z = 4 - 2x.$$

A graph is given on the right to help you. Do **not** evaluate the integral.



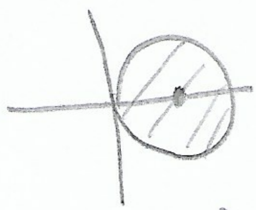
D: intersect the surfaces

$$4 - x^2 - y^2 = 4 - 2x$$

$$0 = x^2 - 2x + y^2$$

$$1 = (x^2 - 2x + 1) + y^2$$

$$1 = (x-1)^2 + y^2$$



polar: $r = 2\cos\theta$

$$\iint_D 4 - x^2 - y^2 - (4 - 2x)\, dA$$

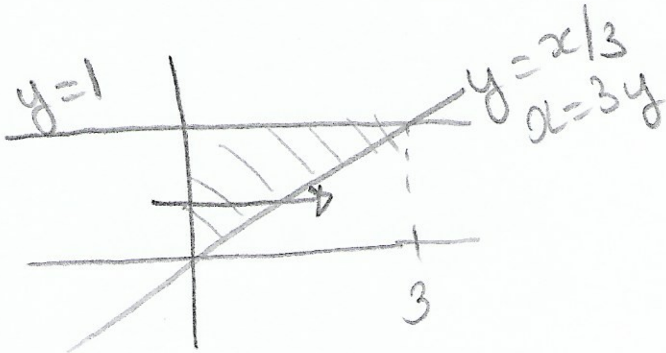
$$= \int_0^{\pi} \int_0^{2\cos\theta} (2r\cos\theta - r^2)\, r\, dr\, d\theta$$

2. (10 points) Evaluate the integral

$$\int_0^3 \int_{x/3}^1 x^2 \sqrt{3+y^2}\, dy\, dx$$

by switching the order of integration.

region $\frac{x}{3} \le y \le 1 \quad , 0 \le x \le 3$



$$\int_0^1 \int_0^{3y} x^2 \sqrt{3+y^2}\, dx\, dy = \int_0^1 \frac{x^3}{3} \sqrt{3+y^2} \Big|_0^{3y} dy$$

$$= \int_0^1 9y^3 \sqrt{3+y^2}\, dy \qquad u = 3+y^2 \quad du = 2y\, dy$$

$$= \frac{9}{2} \int_0^1 (u-3)\sqrt{u}\, du = \frac{9}{2} \int_3^4 u^{3/2} - 3u^{1/2}\, du$$

$$= \frac{9}{2} \left(\frac{2u^{5/2}}{5} - 2u^{3/2} \right) \Big|_3^4 = \frac{9}{2} \left[\left(\frac{2}{5} 4^{5/2} - 2 \cdot 4^{3/2} \right) - \left(\frac{2}{5} 3^{5/2} - 2 \cdot 3^{3/2} \right. \right.$$

$$= \frac{9}{2} \left[\left(\frac{64}{5} - 16 \right) - \left(\frac{18}{5}\sqrt{3} - 6\sqrt{3} \right) \right] = \frac{9}{2} \left[\frac{-16}{5} + 12\sqrt{3} \right]$$

$$= \frac{-72}{5} + 54\sqrt{3}$$

3. (13 points) Find the absolute maximum and absolute minimum values of the function

$$f(x, y) = x^3 - 3xy^2 + 2xy$$

on the closed and bounded domain $x \geq 0$, $y \geq 0$ and $x + y \leq 1$. You may round your answers to two digits after the decimal

critical points:

$f_x = 3x^2 - 3y^2 + 2y = 0$

$f_y = -6xy + 2x = 0$

$2x(1-3y) = 0$

$x = 0$ OR $y = 1/3$

$f_x = -3y^2 + 2y = 0$

$y(-3y+2) = 0$

$y = 0$ or $y = 3/2$

$f_x = 3x^2 - \frac{1}{3} + \frac{2}{3} = 3x^2 + \frac{1}{3} = 0$

no solution

$\boxed{(0,0), (0,3/2)}$

Boundary

$x = 0$, $0 \leq y \leq 1$: $g(y) = 0$

$y = 0$, $0 \leq x \leq 1$: $f(x) = x^3$, $f' = 3x^2 = 0$ $\quad (0,0), \boxed{(1,0)}$

$y = 1-x$, $0 \leq x \leq 1$:

$f(x) = x^3 - 3x(1-x)^2 + 2x(1-x)$

$f' = 3x^2 - 3(1-x)^2 - 3x \cdot 2(1-x)(-1) + 2 - 4x$

$= \cancel{3x^2} - 3 - \cancel{3x^2} + \cancel{6x} + 6x - 6x^2 + 2 - 4x$

$= -6x^2 + 8x - 1$

$x = \frac{-8 \pm \sqrt{64-24}}{-12} = \frac{-8 \pm 2\sqrt{10}}{-12}$

$= \frac{4 \pm \sqrt{10}}{6} \approx 1.2$ or 0.14

not in domain

$\boxed{\left(\frac{4-\sqrt{10}}{6}, \frac{2+\sqrt{10}}{6}\right)}$ $\quad \boxed{(0,1)}$ $(1,0)$

$f(0,y) = 0$, $f(1,0) = 1$ max $\quad f\left(\frac{4-\sqrt{10}}{6}, \frac{2+\sqrt{10}}{6}\right) \approx f(0.14, 0.86) \approx -0.07$ min.

4. Let

$$f(x,y) = \sin(xy - 1).$$

(a) (5 points) Compute all first and second partial derivatives for $f(x,y)$.

$f_x = y\cos(xy-1)$ $\quad$ $f_y = x\cos(xy-1)$

$f_{xx} = -y^2\sin(xy-1)$ $\quad$ $f_{yy} = -x^2\sin(xy-1)$

$f_{xy} = \cos(xy-1) - xy\sin(xy-1)$ $\quad$ $f_{yx} = \cos(xy-1) - xy\sin(xy-1)$

[redacted]

(b) (5 points) Use linear approximation at $(1,1)$ to estimate $f(1.1, 1.2)$.

$f(1,1) = \sin 0 = 0$

$f_x(1,1) = 1\cos 0 = 1$

$f_y(1,1) = 1\cos 0 = 1$

$z - 0 = 1(x-1) + 1(y-1)$

$\boxed{z = x + y - 2}$

$f(1.1, 1.2) \simeq 1.1 + 1.2 - 2 = 0.3$