

MIDTERM 2

MATH 126

Last name, first name: _____

Section: _____

Student number: _____

Signature: _____

Please do not start working until instructed to do so.

You have 50 minutes.

Please show your work.

Scientific, but not graphing calculators are OK.

You may use one 8.5 by 11 sheet of handwritten notes.

Problem 1. _____

Problem 2. _____

Problem 3. _____

Problem 4. _____

Problem 5. _____

Total. _____

Problem 1. (7 points total) Consider the curve $x = t^2$, $y = t^3$ from $t = 0$ to $t = 2$.

a. (4 points) Find the length of the curve.

b. (3 points) Set up the integral that gives the surface area found by rotating this curve about the x -axis. Your integral should only involve the parameter t (no x or y). Do not evaluate the integral.

Problem 2. (12 points) Let $f(x, y) = 1 + x \sin(y - x)$.

a. (4 points) Find the following partial derivatives: f_x , f_y , f_{yy} , f_{yx} .

b. (4 points) Find the equation of the plane tangent to the graph of $f(x, y)$ at the point $(2, 2, 1)$.

c. (4 points) Use your answer to **b** to approximate the value of $f(1.9, 2.2)$.

Problem 3. (10 points total) Consider the polar curve given by

$$r = 1 - \sin \theta.$$

Find the value (or the values) of θ corresponding to the highest point (or the highest points, if there is more than one) on the curve. *Highest points are those with the greatest y coordinate.*

Problem 4. (8 points) The radius of the base of a closed tin can is measured to be 40 cm and the height of the can is measured to be 90 cm. The measurements are used to calculate the total surface area of the can. Use linear approximation to estimate the maximum error in the calculated area if the measurement of the radius has a maximum error of 0.4 cm while the measurement of the height has a maximum error of 0.1 cm. (*The surface area of a closed can is $A = 2\pi r^2 + 2\pi rh$.*)

Problem 5. (13 points) The velocity of a particle is given by

$$\mathbf{r} = \langle 2t, 2t^2, 1 \rangle.$$

Find the following objects related to the curve along which the particle is moving, at time $t = 1$:

- a. (3 pts) the unit tangent vector,
- b. (4 pts) the unit normal vector,
- c. (3 pts) the unit binormal vector,
- d. (3 pts) the curvature.

Hint: you can simplify the unit normal vector so that there is no square root.