Math 126, Midterm #2, 11/17/05

Name: _____

Section: _____

Write your final answer to each question in the space provided to the right. Partial credit will be awarded except where otherwise indicated. You must **show your work** to receive credit for a correct answer to partial credit problems.

You may use a non-graphing calculator and one 8 1/2 x 11 sheet of **handwritten** notes.

The vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ are the unit tangent, principal normal and binormal vectors, respectively.

1. Compute the value of $\mathbf{B} \cdot (\mathbf{N} \times \mathbf{T})$.

_____ *(5)*

No partial credit

2. Consider any point along a trajectory. The line along which the osculating and normal planes intersect at this point is parallel to which of the following vectors?

   - $\mathbf{T}$?
   - $\mathbf{N}$?
   - $\mathbf{B}$?

_____ *(5)*

No partial credit

3. Find the slope of the tangent line to the following polar curve at $\theta = \pi/4$.

$$r = a\cos\theta + b\sin\theta.$$

(10)

4. Consider the curve specified by

$$\mathbf{r}(t) = \left(\frac{1}{2}e^{2t}, \sqrt{2}e^{t}, t\right).$$

Find the length of this curve between $t = 0$ and $t = 3$. *(10)*

5. Let

$$\mathbf{r}(t) = (\sin t, \cos t, t).$$

Find the vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ to this curve. *(10)*

6. You spot a bird on the ground 30m away and consider the problem faced by your ancestors of how to fell it for dinner with a stone. Assuming you throw the stone at 20m/sec, at what **angle** relative to the horizonal must you throw the stone to hit the bird?

To simplify the problem, assume that you release the stone at ground (bird) level. You **must derive all equations** to receive any credit for this problem. The acceleration due to gravity is equal to 9.8m/sec^2. *(10)*