

Math 126, Midterm #2, 02/23/06

Name: _____
Section: _____

Write your final answer to each question in the space provided to the right. Partial credit will be awarded except where otherwise indicated. You must **show your work** to receive credit for a correct answer to partial credit problems.

You may use a non-graphing calculator and one 8 1/2 x 11 sheet of **handwritten** notes.

The vectors **T**, **N** and **B** are the unit tangent, principal normal and binormal vectors, respectively.

1. Let

$$f(x) = \int_0^x \frac{t}{1+t^8} dt.$$

Find the *first three non-zero terms* of the Taylor series expansion for $\frac{df}{dx}$ about $x = 0$.

_____ (10)

2. Consider the polar curve given by

$$\theta = \frac{\pi}{2} \sin r.$$

_____ (10)

(a) Find $\frac{dy}{dx}(r)$.

(b) What is the slope to this curve when $r = \pi/2$?

3. Let

$$\begin{aligned}x &= \sin^2 t, \\y &= \sin 3t.\end{aligned}$$

Find the surface area in the form of an integral swept out by rotating this curve about the x -axis for $0 \leq t \leq \pi/3$. **Do not evaluate the integral.**

_____ (10)

4. Consider the trajectory

$$\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \sin tk.$$

Find the equation of the **normal plane** to the trajectory at $t = \pi/4$.

_____ (10)

5. Let $\mathbf{r}(t)$ describe the trajectory of some object, and let $\mathbf{v}(t)$ and $\mathbf{a}(t)$ be the velocity and acceleration of this object. Find the vector $(\mathbf{a} \times \mathbf{v}) \times \mathbf{v}$ in terms of the vectors \mathbf{T} , \mathbf{N} , and \mathbf{B} , i.e., find α , β and γ such that

$$(\mathbf{a} \times \mathbf{v}) \times \mathbf{v} = \alpha \mathbf{T} + \beta \mathbf{N} + \gamma \mathbf{B}.$$

Hint: Express \mathbf{v} and \mathbf{a} in terms of \mathbf{T} , \mathbf{N} and \mathbf{B} .

_____ (10)