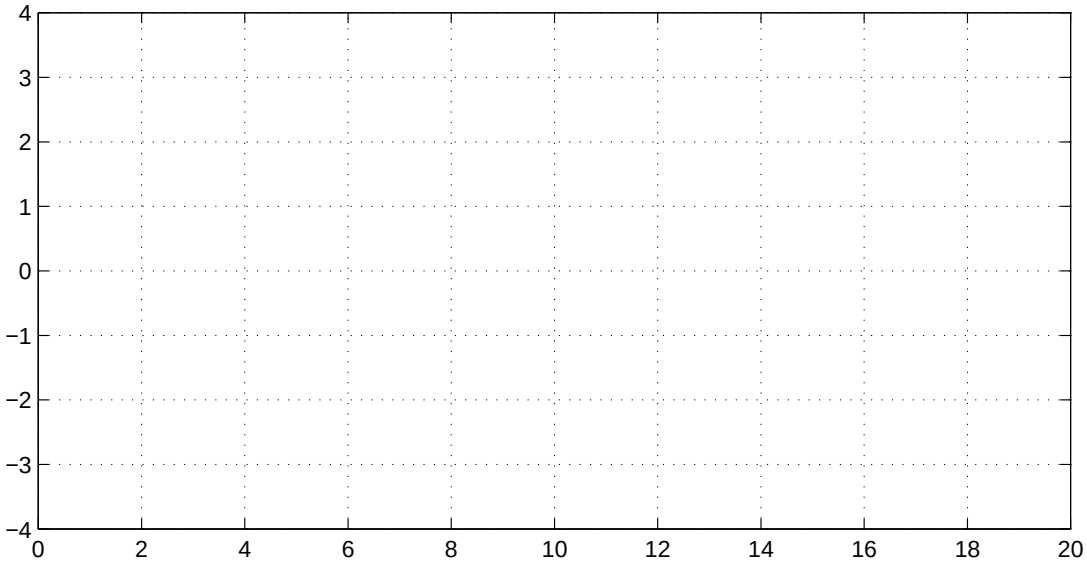
Math 307A Exam 2

Name: _____

1. Solve the initial value problem, write the solution as a product of a sine and a cosine, and plot the solution, showing any interesting phenomena.

$$y'' + 25y = -9\sin(4t)$$
$$y(0) = 0 \qquad y'(0) = -1$$



2. Write down the general form that you would use to find solutions to the following differential equatons. Include both the particular and homogeneous parts, but don't actually solve for the constants.

   - $y'' + 3y' + 2y = \cos(3t) + t\cos(t)$

   - $y'' + 2y' + y = \mathrm{e}^{-t}$

3. A shock absorber coil spring system for an imported automobile is designed to support 350kg. The spring has a constant of 148,750 kg/sec^2 (148750 = 350 x 425). The shock absorber exerts a damping force that is equal to 3500 times the instantaneous velocity of the system (in cm/sec). The system is jarred from equilibrium by a bump which imparts an instantaneous velocity of 75 cm/sec upward (i.e. $y(0) = 0, y'(0) = 75$).

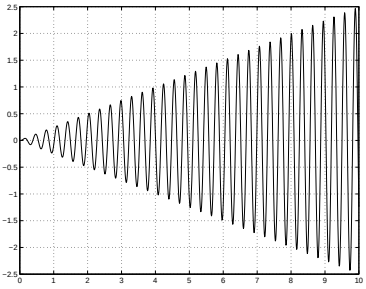
   - Model the system with a differential equation and find a formula for the solution.
   - Is the system underdamped, overdamped or critically damped.
   - Will it ever get more than 5cm high ? More than 1cm high?
   - Roughly, how long before it stays within .1 cm of equilibrium (.1 second, 1 second, 10 seconds) ?

4. Find the steady state solution to

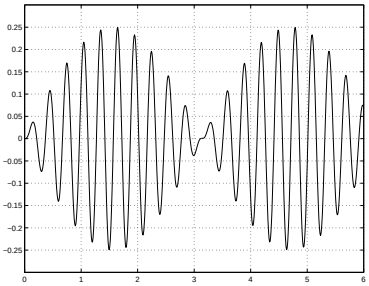
$$y'' + 2y' + 5y = \cos \omega t$$

Express the solution in the amplitude-phase form. At what frequency is the amplitude largest. What is the phase at that frequency?

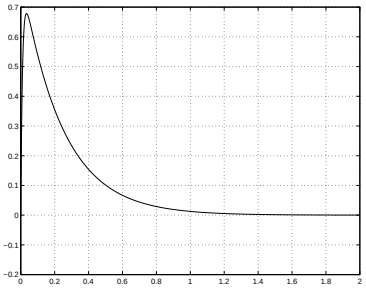
5. To the left of each plot, write the number corresponding to the matching equation. In addtion, write one of the foloowing five words, next to each plot: *beats, resonance, underdamped, overdamped, forced and damped.* Each applies to exactly one plot.

(a) 

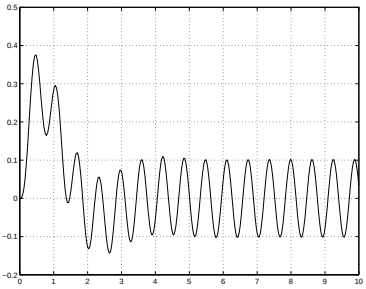
(1) $y'' + 2y' + 4y = -10\cos(10t)$

(b) 

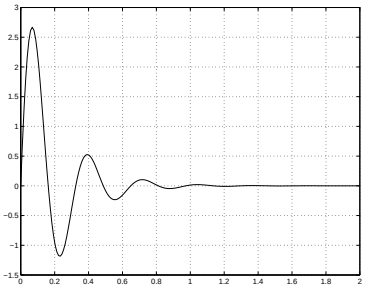
(2) $y'' + 10y' + 400y = 0$

(c) 

(3) $y'' + 400y = -10\cos(20t)$

(d) 

(4) $y'' + 400y = -10\cos(22t)$

(e) 

(5) $y'' + 100y' + 400y = 0$