Final Exam Math 135 Winter 2016

Name:

Student Number:

1. Suppose the path of a particle as a function of time is given by

$$r(t) = (x(t), y(t), z(t)) = (\cos t, \sin t, t^2).$$

As a function of t, find:

a. The velocity vector

b. The acceleration vector

c. The speed

d. $s(t)$ = the distance travelled from time 0 to time t.

(you may leave your answer as an integral).

e. The curvature κ

f. The unit tangent vector T

g. The binormal B

h. The principal normal N

i. The angle between the osculating plane and the x, y plane.

(recall that the angle between two planes equals the angle between their normal vectors).

2. Using Laplace transforms, solve the initial value problem:

$$y'' + 4y' + 13y = e^{-t^2},$$

with $y(0) = 0$ and $y'(0) = 0$. You may leave your answer as an explicit integral.

3. a. Find the recurrance relation for the coefficients of the power series solution around $x_0 = 0$ for the differential equation

$$(1 + x + x^2)y'' - 2xy' + 12y = 0, \tag{1}$$

where $y(0) = 1$ and $y'(0) = 2$.

b. Use your answer to find the Taylor polynomial (about $x_0 = 0$) of degree 4 for y.

c. Give an interval on which the series solution in part a is guaranteed to converge.

d. Use our existence and uniqueness theorem to prove that there are two solutions y_1 and y_2 to equation (1) which are power series expansions about the point $x_0 = 5$ (instead of $x_0 = 0$) satisfying $y_1(5) = 1$, $y_1'(5) = 0$, $y_2(5) = 0$ and $y_2'(5) = 1$, and find an explicit disk centered at $x_0 = 5$ on which these power series converge.

e. Prove that there exists constants c_1 and c_2 so that

$$y = c_1 y_1 + c_2 y_2$$

on an interval containing 0, where y is the solution from part a and y_1 and y_2 are the solutions in part d.