Math 135 Final Review

Review Questions on vectors, 3D space and vector calculus

1. Given the two vector functions $\mathbf{r_1}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$ and $\mathbf{r_2}(t) = t^2\,\mathbf{i} + t^3\,\mathbf{j} + t^4\,\mathbf{k}$

   (a) Find the point where the curves traced by the two vector functions intersect.

   (b) Find parametric equations for each of the tangent lines to these curves at their point of intersection.

   (c) Find the angle between the two tangent lines to the curves at that point.

   (d) Find the equation of the plane containing these two tangent lines.

2. Define a curve by
$$\mathbf{r}(t) = \cos 3t\,\mathbf{i} + t\,\mathbf{j} - \sin 3t\,\mathbf{k}.$$
   Find the vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ as functions of t.

3. Consider the plane Π: $x + 2y + 3z = 10$

   (a) Show that the line ℓ given by the equation $\mathbf{r}(t) = (4\mathbf{i} + 3\mathbf{j}) + t(3\mathbf{j} - 2\mathbf{k})$ is contained in the plane Π.

   (b) Find a parametric equation for the line in the plane Π passing through the point $P(3, 2, 1)$ on the plane and intersecting the line ℓ orthogonally.

4. Given the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ and the plane plane $x + y + 2z = 4$ at a point.

   (a) Find the point where the curve intersects the plane.

   (b) Find the angle between the tangent to the curve and the normal to the plane at that point.

5. Three objects move in space according to the equations
$$\mathbf{r} = \mathbf{r}_1(t) \quad \mathbf{r} = \mathbf{r}_2(t) \quad \text{ and } \mathbf{r} = \mathbf{r}_3(t)\,,$$
   where t denotes time. Let $A(t)$ denote the area of the triangle formed by the three objects. Suppose that
$$\begin{array}{lll} \mathbf{r}_1(0) = \mathbf{i} + \mathbf{j} + \mathbf{k} & \mathbf{r}_2(0) = \mathbf{i} + \mathbf{j} - \mathbf{k} & \mathbf{r}_3(0) = \mathbf{k} \\ \mathbf{r}_1'(0) = \mathbf{i} & \mathbf{r}_2'(0) = \mathbf{j} & \mathbf{r}_3'(0) = \mathbf{k} \end{array}$$
   Compute $A'(0)$.

6. Given the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ and the sphere $x^2 + y^2 + z^2 = 3$,

   (a) Find the point where the curve intersects the sphere.

   (b) Find the angle between the tangent to the curve and the normal to the sphere at that point.

   (c) Find the equation of the plane tangent to the sphere $x^2 + y^2 + z^2 = 3$ at that point.

7. The trajectory of an object is given by the formula $\mathbf{r}(t) = t\,\mathbf{i} + \mathbf{j} + \dfrac{t^2}{2}\mathbf{k}$, where t denotes time.

   (a) Find the speed of the object at time t.

   (b) Find the unit tangent vector $\mathbf{T}$ to the curve traversed by the object as function of t.

   (c) Find both the curvature κ and the unit normal $\mathbf{N}$ to the curve traversed by the object as functions of t.

8. Use vectors to find the distance from the point $P(1, 2, 3)$ to the line $\mathbf{r}(t) = (2 - 4t)\mathbf{i} + (-1 + 3t)\mathbf{j} + 5t\mathbf{k}$