- Routine Problems:

  §14.1: #12 #16 #57;
  §14.2: #36;
  §14.3: #33 #34 #39;
  §14.4: #5 #19;
  §14.5: #2 #35 #44 #50 #52.

  Page 747: #28 #29 #35 #38 #46 #48.

- To hand in:

  (1) A vector-valued function $\mathbf{G}$ is called an *antiderivative* for $\mathbf{f}$ on $[a, b]$ provide that $\mathbf{G}$ is continuous on $[a, b]$ and $\mathbf{G}'(t) = \mathbf{f}(t)$ for all $t \in (a, b)$.

  (a) Show that if $\mathbf{f}$ is continuous on $[a, b]$ and $\mathbf{G}$ is an antiderivative for $\mathbf{f}$ on $[a, b]$ then

  $$\int_a^b \mathbf{f}(t)\, dt = \mathbf{G}(b) - \mathbf{G}(a)$$

  (b) Show that if $\mathbf{f}$ is continous on $[a, b]$ and $\mathbf{F}$ and $\mathbf{G}$ are antiderivatives for $\mathbf{f}$ on $[a, b]$ then

  $$\mathbf{F} = \mathbf{G} + \mathbf{C}$$

  for some constant vector $\mathbf{C}$.

  (2) The force due to gravity is given by the constant vector

  $$\mathbf{F} = -mg\,\hat{\mathbf{k}}\,,$$

  where we have chosen a coordinate system where $-\hat{\mathbf{k}}$ points toward the center of the Earth.

  Let $\mathbf{r} = \mathbf{f}(t)$ denote the trajectory of an object of mass m moving under the influence of gravity, with no air resistance. Let $\mathbf{r}_0$ and $\mathbf{v}_0$ denote the position and velocity of the object at time $t = 0$. Show that

  $$\mathbf{f}(t) = \mathbf{r}_0 + t\,\mathbf{v}_0 - \frac{1}{2}gt^2\,\hat{\mathbf{k}}\,.$$

  Hint: *Recall that* $m\mathbf{f}''(t) = \mathbf{F}$.

  (3) Three objects move in space according to the equations

  $$\mathbf{r} = \mathbf{r}_1(t) \quad \mathbf{r} = \mathbf{r}_2(t) \quad \text{and } \mathbf{r} = \mathbf{r}_3(t)\,,$$

  where t denotes time. Let $A(t)$ denote the area of the triangle formed by the three objects. Suppose that

  $$\begin{array}{lll} \mathbf{r}_1(0) = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} & \mathbf{r}_2(0) = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}} & \mathbf{r}_3(0) = \hat{\mathbf{k}} \\ \mathbf{r}_1'(0) = \hat{\mathbf{i}} & \mathbf{r}_2'(0) = \hat{\mathbf{j}} & \mathbf{r}_3'(0) = \hat{\mathbf{k}} \end{array}$$

  Compute $A'(0)$.

(4) Let $\mathbf{r} = \mathbf{r}(s)$ be the arc length parametrization of a simple curve. Suppose that $\|\mathbf{r}(s)\| = 1$ for all s and that $\mathbf{r}(s)$ has continuous first and second derivatives.

(a) Show that the three vector-valued functions $\mathbf{r} = \mathbf{r}(s)$, $\mathbf{T} = \mathbf{T}(s) =: \frac{d\mathbf{r}(s)}{ds}$, and $\mathbf{U} = \mathbf{U}(s) =: \mathbf{r}(s)\times\mathbf{T}(s)$ form an oriented frame (i.e. that they are mutually orthogonal unit vectors and that the triple scalar product $(\mathbf{r}\times\mathbf{T}) \cdot \mathbf{U}$ is positive).

(b) Show that there is a scalar function $\beta(s)$ such that the following equations are satisfied:

$$\frac{d\mathbf{r}}{ds} = \mathbf{T}\,, \quad \frac{d\mathbf{T}}{ds} = -\mathbf{r} + \beta\mathbf{U}\,, \quad \frac{d\mathbf{U}}{ds} = -\beta\mathbf{T}\,.$$

(c) Now let $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$ denote the Frenet frame for the curve. Show that

$$\mathbf{N} = \frac{-\mathbf{r} + \beta\mathbf{U}}{\sqrt{1+\beta^2}}$$

and from that conclude that $\kappa = \sqrt{1+\beta^2}$.

(d) Finally, use part (c) to find a formula for the torsion τ of the curve in terms of β and its derivative β'. From this, conclude that $\tau(s) = 0$ if and only if $\beta'(s) = 0$.