These problems should be attempted before class on Tuesday.

Routine problems (not to hand in):

12.9: 1, 6, 7, 9, 19

series handout: 1,2,3,4,5

Turn in :

1. series handout #6 (Hint: f behaves like the first non-zero term in its power series expansion near 0 - see problem 4.)

2. Show that $\displaystyle\sum_{k=0}^{\infty} \frac{\sin k}{2^k}$ converges. Evaluate it exactly by using the fact that $\sin k = \mathrm{Im}(e^{ik})$. ($\mathrm{Im}(a+ib) = b$, the imaginary part of $a+ib$.)

Hint: *It will be useful to recall that* $\frac{1}{a+ib} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$, *($a, b$ real).*

3. Prove the following:

a. The radius of convergence of the power series for $1/(z-a)$ centered at b is the distance from b to a. Here a and b are complex numbers.

b. The radius of convergence of the power series for $(z-a)^{-k}$, $k = 2, 3, ...$ centered at b is also equal to the distance from b to a.

c. Definition: A rational function r is a function of the form $r = p/q$ where p and q are polynomials. Prove that a rational function can be written in the form $r = p/q$ where p and q have no common zeros. Hint: fundamental theorem of algebra.

d. Prove that the radius of convergence of the power series for r centered at b is the distance from b to the nearest zero of q. Hint: Use Corollary 3.3 from the Complex Numbers handout last quarter. Part of your proof is that the series does not converge on a disk centered at b of larger radius.