1. Let $S$ be the set of numbers of the form $(-1)^{n^{2}} \frac{n+1 / n!}{n+1}$ for $n \geq 2$. Explain why $S$ does or does not have a least upper bound. If it has a least upper bound, what is it? Answer the same question about greatest lower bounds.
2. For each integer $n \geq 1$, let $a_{n}=2 \ln (3 n-1)-\ln \left(2 n^{2}+2 n+3\right)$. Does the sequence $\left\{a_{n}\right\}$ converge or diverge? If it converges, what is the limit?
3. Consider the sequence $\left\{a_{k}\right\}$ defined as follows:

$$
a_{0}=1 \quad a_{n+1}=1-a_{n} / 2 \text { for } n=0,1,2, \ldots
$$

Show that the sequence converges to $2 / 3$.
4. Consider the sequence $\left\{a_{k}\right\}$ defined by $a_{0}=0, a_{n+1}=1+m a_{n}$, where $m$ is a real number with $|m|<1$. Does the series converge? Explain your answer. If the series does converge, what is its limit?
5. Test the following series for convergence, and (where appropriate) test for absolute and conditional convergence.
(a) $\sum_{k=1}^{\infty} \frac{k!}{k^{k}}$
(b) $\sum_{n=1}^{\infty} \frac{k^{k}}{k!}$
(c) $\sum_{n=1}^{\infty}(-1)^{k} \frac{k+2}{k^{2}+k}$
(d) $\sum_{k=1}^{\infty}(-1)^{k} \frac{k+2}{k^{3}+k}$
(e) $\sum_{k=0}^{\infty} \frac{k \sin (k \pi)}{k^{2}+1}$
(f) $\sum_{k=1}^{\infty}(-1)^{k} \frac{\ln k}{k^{3}+\ln k+1}$
(g) $1+\frac{1 \cdot 2}{1 \cdot 3}+\frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5}+\frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7}+\ldots$
6. Evaluate the following limits in any way you wish.
(a) $\lim _{x \rightarrow 0} \frac{x \sin \left(x^{2}\right)-\sin \left(x^{3}\right)}{\sin \left(x^{7}\right)}$
(b) $\lim _{x \rightarrow 0} \frac{\cosh x-\cos x}{\sin x^{2}}$
(c) $\lim _{x \rightarrow 0} \frac{\cosh x-\cos x}{x^{2}}$
(d) $\lim _{x \rightarrow 0} \frac{\cos x-\cos 2 x}{x \sin 4 x}$.
7. Evaluate the integrals or explain why you can't.
(a) $\int_{-1}^{1} \frac{x}{\sqrt{1-x^{2}}} d x$
(b) $\int_{-1}^{+1} \frac{2 x}{1-x^{2}} d x$
(c) $\int_{0}^{1} \frac{d x}{x^{2 / 5}}$
(d) $\int_{-1}^{1} \frac{x}{\sqrt{1-x^{2}}} d x$
(e) $\int_{-\infty}^{\infty} \frac{x}{1+x^{2}} d x$
8. For each of the following power series: (i) find the radius of convergence, (ii) find the interval of convergence, (iii) determine the values of $x$ for which the series is absolutely convergent and the values of $x$ for which the series is conditionally convergent.
(a) $\sum_{k=1}^{\infty} \frac{2^{k} \ln (k+1)}{k} x^{k}$.
(b) $\sum_{k=0}^{\infty} \frac{x^{2 k+1}}{3^{k} \ln (2 k+2)}$.
(c) $\sum_{k=0}^{\infty} \frac{2^{k} x^{k}}{k+1}$.
9. Use the formula $\frac{d}{d x}\left(\tanh ^{-1} x\right)=\frac{1}{1-x^{2}}$, valid for $-1<x<1$, to compute the Taylor series for $\tanh ^{-1} x$. What is the interval of convergence for the series?
10. Find the first three non-zero terms in the series expansion of $\arcsin (x)$ about $x=0$.

Hint: Recall that $\arcsin (x)=\int_{0}^{x}\left(1-t^{2}\right)^{-1 / 2} d t$. If you wish, you may use the binomial expansion of $(1+x)^{-1 / 2}$.
11. Give the Taylor series about 0 of the function $f(x)=\int_{0}^{x} \sin \left(t^{2}\right) d t$. For what values of $x$ does the series converge to $f(x)$ ? Justify your answer.
12. Review the problems in the series handout and homework 4.

