For practice, do as many of the problems at the end of each section of chapter 1 in Linear Algebra Done Wrong as you have time to do. Problems (3) and (4) below are generalizations of two of those.

In particular, you should be comfortable with multiplying a matrix with a vector to find the image of the vector, multiplying two matrices to compose two transformations and writing down the matrix of a transformation with respect to a given basis with the columns of the matrix being the images of the basis vectors in the domain.

(1) Let $C^\infty([0,1])$ denote the set of infinitely differentiable functions on the interval $[0,1]$.

Show that $C^\infty([0,1])$ is a vector space under the operations of addition of functions and multiplication by a real number.

(2) Let $V = C^\infty([0,1])$ be the vector space of infinitely differentiable functions on $[0,1]$. Let

$$W = \{f \in V \,:\, f'' + pf' + qf = 0\}$$

for $p \in V$ and $q \in V$.

(a) Prove that W is a subspace of V.

(b) What does the existence and uniqueness theorem for ordinary differential equations tell you about the number of elements in any basis for W?

(c) Suppose that $p(t) = 0$ and $q(t) = 1$. Give two distinct bases for W

(3) Let $\mathbf{T} : \mathbf{R}^2 \to \mathbf{R}^2$ be the **projection** of the points on the xy-plane to the line through the origin given by the equation $\alpha x + \beta y = 0$. Find the matrix of this transformation with respect to the standard basis on $\mathbf{R}^2$.

(4) Let $\mathbf{T} : \mathbf{R}^2 \to \mathbf{R}^2$ be the **reflection** of the points on the xy-plane through the line $\alpha x + \beta y = 0$. Find the matrix of this transformation with respect to the standard basis on $\mathbf{R}^2$.

Bonus Do questions (4) and (5) on $\mathbf{R}^3$ and the plane $\alpha x + \beta y + \gamma z = 0$.