1. Suppose $p(x) = a_0(1 + x) + a_1(x + x^2) + a_2(x^2 + x^3) + a_3(x^3 + 1)$. Find coefficients b_0, b_1, b_2, b_3 so that $p(x) = b_0(1 + x^2) + b_1(x^2 + x^3) + b_2(1 + 3x) + b_3(x^3 + x)$. A natural way to attack this problem without knowledge of linear algebra is to equate coefficients of similar powers of x for the two representations, then solve four equations with four unknowns. Solve the problem using the linear algebra technique for changing bases as given in the text. Then describe how these two methods are related.

2. Let n be odd, and suppose A is an $n \times n$ skew symmetric matrix (i.e. $A^T = -A$). Prove that A does not map R^n onto R^n.

3. A real square matrix Q is orthogonal if $Q^TQ = I$. Prove that if Q is orthogonal then $det(Q) = \pm 1$.

4. (see problem 3.11 in Chapter 3). Find a 4×4 matrix M which can be written in block form with four 2×2 matrices A, B, C, D where A is the upper left block, B is the upper right block, C is the lower left block, and D is the right block such that

$$det(M) \neq det(A)det(D) - det(B)det(C).$$

5. Problem 5.3 page 95