Chapter 4:

(1) Problem 1.6

(2) Problem 1.7

(3) Problem 2.12

(4) Problem 2.13

(5) Consider the linear map $L_A : \mathbb{R}^n \to \mathbb{R}^n$ given by muliplication by the real $n \times n$ matrix A.

Suppose that $\lambda = \alpha + i\beta \in \mathbb{C}$, $\beta \neq 0$, is a complex eigenvalue of L_A, with complex eigenvector $Z = X_1 + iX_2$, where X_1 and X_2 are column vectors in $\mathbb{R}^n$ (not both zero). Thus,

$$A \cdot Z = (A \cdot X_1) + i(A \cdot X_2) = (\alpha X_1 - \beta X_2) + i(\alpha X_2 + \beta X_1)\,.$$

(a) Prove that the vectors X_1 and X_2 are linearly independent and, therefore, span the 2-dimensional subspace $W = \text{span}(X_1, X_2) \subset \mathbb{R}^n$.

(b) Show that L_A restricts to define a linear map $L_W : W \to W$ and that the matrix of L_W with respect to the basis $\{X_1, X_2\}$ of W is

$$|\lambda| \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

where $\lambda = |\lambda|e^{i\theta}$ (the polar form of λ).

(c) What is the geometrical interpretation of the result of part (b)?