Midterm 1 Math 136 Spring 2016

Name:

3. Let $L_A : \mathbb{R}^6 \to R^4$ be given by

$$L_a(\mathbf{x}) = A\mathbf{x},$$

for $\mathbf{x} \in \mathbb{R}^6$, where A is the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 2 & 0 \\ 3 & 4 & 3 & 1 & 2 & 0 \\ 3 & 3 & 3 & 1 & 2 & 1 \end{pmatrix}$$

(a) Find the row reduced echelon form of A.

(b) What is the dimension of $\mathrm{ranL_A}$ (the image or range of L_A)?

(c) What is the dimension of $\mathrm{kerL_A}$ (the kernel of L_A)?

(d) Find a basis for $\mathrm{kerL_A}$.

(e) Find a basis for $\mathrm{ranL_A}$.

2. Show that the equation of the plane $Ax + By + Cz + D = 0$ passing through the three points $(1, 1, 1)$, $(3, 2, 1)$, and $(1, 2, 3)$ can be written in the form

$$\begin{vmatrix} x & y & z & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 1 \\ 1 & 2 & 3 & 1 \end{vmatrix} = 0$$

You do not have to explicitly compute the values of A, B, C, D but rather just describe how to find values, not all of which are zero, and give reasons why your process works.

3. Let V be a vector space of dimension n and let $P : V \to V$ be a linear map satisfying the identity

$$P \circ P = P\,,$$

(i.e. $P(P(\vec{v})) = P(\vec{v})$ for all $\vec{v} \in V$). Suppose that $\{\vec{b}_1, \ldots, \vec{b}_r\}$ is a basis for the range of P and $\{\vec{b}_{r+1}, \ldots, \vec{b}_n\}$ is a basis for the kernel of P.

(a) Show that $\{\vec{b}_1, \ldots, \vec{b}_n\}$ is a basis for V.

(b) Suppose $r = 3$ and $n = 5$, find the matrix of P with respect to the basis $\{\vec{b}_1, \ldots, \vec{b}_5\}$.

4. Suppose that A is an $n \times n$ matrix with only one eigenvalue λ and suppose that the geometric and analytic multiplicity of this eigenvalue agree. What is A? (prove your answer is correct).