Midterm 1 Math 136 Spring 2016

Name:

Show your work. If your solution continues to another page, clearly indicate where, so that I can follow your argument.

1. Let

$$B = \begin{pmatrix} 14 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 11 \end{pmatrix}$$

Give the definition of each term below and a brief explanations for your answers. Although some require computations, they are meant to be minimal and fast. An answer of yes or no without any reason is worth little. Is the matrix:

(i) Self-adjoint?(Yes/No)

(ii) Normal?(Yes/No)

(iii) Diagonalizable?(Yes/No)

(iv) Diagonalizable by a unitary matrix?(Yes/No)

(v) Invertible?(Yes/No)

(vi) Positive definite?(Yes/No)

(vi) Positive semi-definite?(Yes/No)

(vii) Unitary?(Yes/No)

(viii) Orthogonal?(Yes/No)

2. Let

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}$$

Find the matrix $A^+$ so that $x = A^+ b$ is the least square solution to $Ax = b$ of minimal norm.

3. Let $A$ be the matrix given in problem 2. Find matricies $W$, $\Sigma$, $V$ so that $A = W\Sigma V^*$ is the Singular Value Decomposition of $A$. You may use any results you obtained in problem 2.

4. a. Find the Cholesky decomposition of

$$C = \begin{pmatrix} 100 & 90 & 70 & 40 \\ 90 & 145 & 111 & 60 \\ 70 & 111 & 110 & 56 \\ 40 & 60 & 56 & 30 \end{pmatrix}$$

b. Use your answer to part a. to solve $Cx = (0 \;\; 64 \;\; 43 \;\; 24)^*$ for $x$.