MATH CIRCLE WINTER QUARTER WEEK 2

0. More algebra workout problems!

(1) Expand $(x-y)(x^3+x^2y+xy^2+y^3)$.

(2) Write $\dfrac{2+\sqrt{5}}{3-\sqrt{5}}$ in the form $a+b\sqrt{5}$.

Hint. Multiply both the numerator and denominator by $3+\sqrt{5}$.

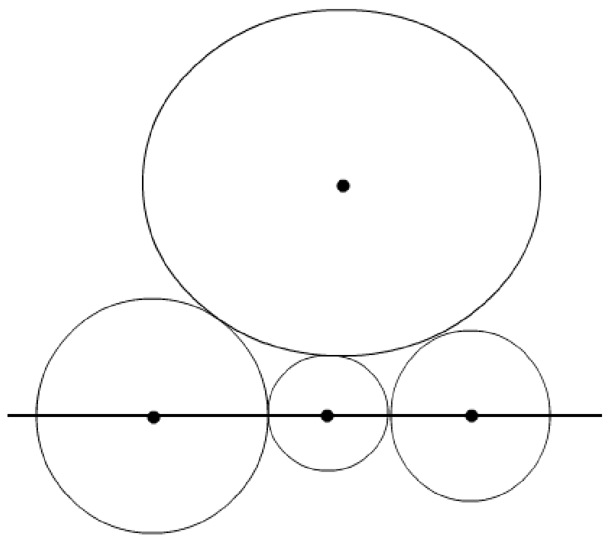
(3) Solve for z in the following equation. *Hint.* Quadratic equation.

$$-p^2+(z^2-z)p=-q^2z+q$$

1. If point O is inside ΔABC, show that $AO+OC\leq AB+BC$.

2. Two villages lie on opposite sides of a river whose banks are parallel lines. A bridge is to be built over the river, perpendicular to the banks. Where should the bridge be built so that the path from one village to the other is as short as possible?

3. Four circles are arranged as shown, each tangent (*just* touching) at the points of intersection. The centers of the three lower circles all lie on a line, and the top circle is tangent to each of those other circles. Show that the radius of the top circle is larger than the radius of at least one of the other circles.



4. Let D be a point on side $\overline{AC}$ of ΔABC.

(a) Show that $BD \leq AB + BC$.

(b) **(Challenge)** Show that $BD \leq \max(AB, BC)$.
(the max of two numbers is just the larger one)

(c) Is it true that $BD \geq \min(AB, BC)$?
(the min of two numbers is just the smaller one)