MATH CIRCLE HOMEWORK 8

1. Dylan has ten line segments. Is it true that there must be three segments which form a triangle?

2. Suppose that we color the entire two-dimensional plane using **two** different colors. That is, every point (x, y) is painted one of the two colors. Show that there are must be two points of identical color exactly one meter apart.

3. Determine whether there are any **integers** x, y, z, and t which satisfy the equation $x^2 + y^2 + z^2 = 8t - 1$.

4. Two players take turns placing X's and O's on a 9×9 board. The first player places X's and the second places O's. Once the board is filled, the first player gets a point for every row or column where there are more X's than O's and the second player gets a point for every row or column where there are more O's than X's. The player with the most points wins; who has the winning strategy and what is it?

5. Find the remainder when 2^{100} is divided by 101, and when 8^{900} is divided by 29.

6. Prove that for each prime p the difference

$$111...11222...22333...33...888...88999...99 - 123456789$$

(where, in the first number, each non-zero digit is written exactly p times) is divisible by p.