

Math Circle - Homework 3

The first few problems build on the Hilbert's Hotel worksheet from class (available online). Hilbert has just inherited a new hotel, *The Real Deal Hotel*. This new hotel has many more rooms than his original hotel. Indeed, in *The Real Deal Hotel* there is a room number for every single **positive real number between 0 and 1 (not including either 0 or 1)**.



1. Hilbert's original *Happening Hotel of Infinity* is completely full, with a customer in each room 1, 2, 3, 4, etc. He wants to move them into his new (empty) *Real Deal Hotel*. Help Hilbert by figuring out a formula for doing this (i.e. if a person is in room n of the *Happening Hotel*, what should that person's room number be in the *Real Deal Hotel*?).
2. The new *Real Deal Hotel* is all booked up for the night, but that pesky astronaut from Pluto has shown up again. He begs and pleads *again* for a room in the hotel. In his original hotel, Hilbert was able to help out the Plutonian without kicking any guests out in the cold, just by shifting some of them around. Can Hilbert similarly make room for the astronaut in this new *Real Deal Hotel*? If so, how? If not, why not?

3. Recall the Betridosians from the original Hilbert's Hotel worksheet. These are the aliens for which there is a unique Betridosian for every real number. There is a group on the planet Betridos called the *Positives*. As you may have already guessed, the *Positives* consists of all Betridosians who have a number which is strictly greater than 0.

The Real Deal Hotel is empty, and the *Positives* all want their own room in this new hotel. Can you help them out?

4. The rate for staying at Hilbert's hotel is one bar of silver per week. However, Hilbert makes his guests pay for their room daily. The Plutonian astronaut will be staying exactly one week, and he luckily has a single silver bar. However, this is a full silver bar, and the Plutonian needs to pay Hilbert seven times!

As are all silver bars, this one is segmented into seven equal segments. The astronaut has a special tool which will break a silver bar into two pieces, but unfortunately the tool may only be used twice before evaporating into oblivion. How can the Plutonian traveler go about paying Hilbert daily?

