Math Circle - Homework 4

1. (**10 points**) For any positive integer $n > 5$, show how to decompose a square into n sub-squares (possibly of different sizes).

2. (**10 points**) Andrey and Bethany are gambling on a few games of chance in the *Inductive Casino*. They take turns playing a single game each. In the first game, Andrey wins \$1! But in the second game, Bethany loses \$2. In the third game, Andrey is back to winning – this time \$3. However in the fourth game Bethany loses \$4. They keep on this same track all night, where in any odd-numbered game number $2n - 1$, Andrey wins $2n - 1$ dollars. But directly after that, Bethany loses $2n$ dollars in the next gamble.

After the first game, their winnings are \$1. After the second, they have a net (total) loss of \$1. After the third game, Andrey's win brings up their net (total) winnings to \$2 = 1 - 2 + 3. However Bethany's loss of \$4 in the fourth game gives them now a net loss of \$2.

After every odd-numbered game $2n - 1$, Andrey and Bethany have a net winning. And after every even-numbered game $2n$, they have a net loss. What are these total winning and loss values (in terms of n)? Prove that both your winning and loss values are correct.

3. (**10 points**) Prove **using mathematical induction** that there are $\dfrac{n(n - 1)}{2}$ ways of choosing *two different socks* from a drawer full of n socks.

4. (**10 points**) Is the value of $n^2 + n + 41$ prime for all nonnegative integers $n = 0, 1, 2, 3, \ldots$? If so, prove it. If not, why not?