Math Circle - Homework 6

1. Finish the problems from *Hacking* and *Generating Secret Codes* worksheets.

2. (**10 points**) Explain how you could create a formal grammar which generates every word in the entire English language, and no other words.

3. (**10 points**) Suppose you have two formal grammars G' and G'' with start states S' and S'', respectively. The languages generated by these grammars are $L(G')$ and $L(G'')$. Explain how to create a new grammar G with start state S which has $L(G) = L(G') \cup L(G'')$. That is, the language generated by G should be exactly those words which can be generated either by G' or G'' (or both).

Use this idea to create a formal grammar which generates the language

$$L = \{a^n : n \text{ is a multiple of at least one of 3 or 5}\}.$$

4. (**10 points**) A race of alien beings has only two characters in their written alphabet $\Sigma = \{a, b\}$. Of course, it is not the case that every random string of as and bs forms a word in their alien language. In fact, their language has quite an elegant description, as outlined below.

For any arbitrary string $w \in \Sigma^*$, let w° denote the string where all the as are replaced by bs, and vice-versa. We call w° the *switch up* of the string w. For example, $(bba)^\circ = aab$, and $(aabbaba)^\circ = bbaabab$. Recall also that w^R denotes w "spelled backwards" or "reversed."

Using this notation, the alien language is perfectly described as

$$L = \{w \in \Sigma^* : w = (w^R)^\circ\}.$$

This is the language consisting of all words which are the same after reversing them and then switching them up. Some examples of words in L are ab and $aababb$. What are five more examples of alien words?

(i) What can you say about the length of any word in the alien language?

(ii) Create a formal grammar which generates the language L.