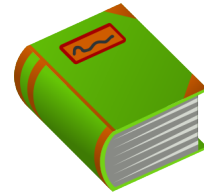


Math Circle - Homework 6

1. Finish the problems from *Hacking* and *Generating Secret Codes* worksheets.

2. (10 points) Explain how you could create a formal grammar which generates every word in the entire English language, and no other words.



3. (10 points) Suppose you have two formal grammars G' and G'' with start states S' and S'' , respectively. The languages generated by these grammars are $L(G')$ and $L(G'')$. Explain how to create a new grammar G with start state S which has $L(G) = L(G') \cup L(G'')$. That is, the language generated by G should be exactly those words which can be generated either by G' or G'' (or both).

Use this idea to create a formal grammar which generates the language

$$L = \{a^n : n \text{ is a multiple of at least one of } 3 \text{ or } 5\}.$$

4. (10 points) A race of alien beings has only two characters in their written alphabet $\Sigma = \{a, b\}$. Of course, it is not the case that every random string of a s and b s forms a word in their alien language. In fact, their language has quite an elegant description, as outlined below.

For any arbitrary string $w \in \Sigma^*$, let w° denote the string where all the a s are replaced by b s, and vice-versa. We call w° the *switch up* of the string w . For example, $(bba)^\circ = aab$, and $(aabbaba)^\circ = bbaabab$. Recall also that w^R denotes w “spelled backwards” or “reversed.”

Using this notation, the alien language is perfectly described as

$$L = \{w \in \Sigma^* : w = (w^R)^\circ\}.$$

This is the language consisting of all words which are the same after reversing them and then switching them up. Some examples of words in L are ab and $aababb$. What are five more examples of alien words?

- (i) What can you say about the length of any word in the alien language?
- (ii) Create a formal grammar which generates the language L .

