

Things to Think on Week 1

1. Your 3 year old siblings spills crumbs all over your living room; you count and there are 23 of them. The room is $6m \times 4m$. Can you find a square meter free of crumbs?

2. The country of Mathopia has seven cities, one for each Millenium problem (if you don't know what these are look them up!). Mathiopian lore tells of a traveler who left Riemannia (one of the cities of Mathiopia), visited each of the other cities an even amount of times each, and returned back to Reimannia having traversed exactly 17 roads. As the local mythbuster, your goal is to either show how this is possible (by drawing a possible map of Mathiopia and showing the travelers path) or show why this is not possible. If you find that the travelers tale is only a myth, suggest a correction that would make it more believable.

3. You have a square dartboard with sides of length $50cm$. You throw five darts that hit the board. Prove that you can find two of them that are within 36 cm of each other.

4. Show that for any positive integer n ,

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

5. A $3 \times n$ recatngle has n blue, n green, and n red checkers on it. Prove that you can always rearrange the checkers so that each column has exactly one of each color.

6. The country of Canaria uses fenniks for currency. Their government only prints bills that worth 3 fenniks and 5 fenniks. Prove that any purchase of over 7 fenniks can be paid entirely without needing change. (E.g. you can pay 13 fenniks with two 5's and a 3.)