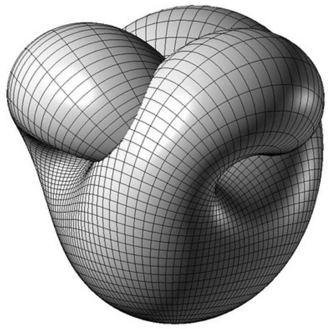
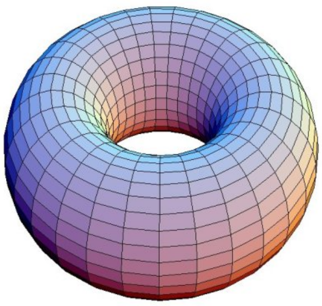
Math Circle - Spring 2012 - Homework 3

For problems 1-4, let $S = \mathbb{R}^2 = \{(x, y) : x \text{ and } y \text{ are real numbers}\}$. For the equivalence relation $\sim$ defined, draw the quotient set $S/\sim$ as a two-dimensional shape with possible edges identified with proper orientation. Then try to recognize $S/\sim$ as some spacial shape. Draw this shape in three-dimensional space if you can.

1. (10 points) $(x, y) \sim (a, b)$: $x - a$ is an integer **and** $y = b$.

2. (10 points) $(x, y) \sim (a, b)$: $x - a$ is an integer **and** $y - b$ is an integer.





3. (10 points) $(x, y) \sim (a, b)$: $(x^2 + y^2) - (a^2 + b^2)$ is an integer.

4. (10 points) $(x, y) \sim (a, b)$: either (i) there exists $\lambda > 0$ such that $x = \lambda a$ and $y = \lambda b$ **or** (ii) $x = a = 0$.

5. (10 points) Is the following relation $\propto$ on $\mathbb{R}^2$ *transitive*?

$(x, y) \propto (a, b)$: either (i) $(x, y) = (a, b)$ **or** (ii) the line through the two distinct points (x, y) and (a, b) does not go through the origin $(0, 0)$.