Things to Think on Week 4

Warm-up : All the students in a school are arranged in a rectangular array. After that, the tallest student in each row is picked, and John is the shortest of these. Then the shortest person in each column was picked and Mary was the tallest of these. Who is taller, John or Mary?

1. The numbers 1 through 64 are written in squares of a chessboard (each number appearing exactly once). Prove that there are two neighboring squares that differ by at least 5.

2. Prove the following identity:

$$\binom{2n+2}{n+1} = \binom{2n}{n+1} + 2\binom{2n}{n} + \binom{2n}{n-1}$$

   (Hint: Don't try to calculate anything... draw some pictures and use your brain.)

3. (a) If your order n hats from a store that sells m different kinds of hats, then there are how many different possible orders you could make?

   (b) Prove that

$$\binom{n+m-1}{n} = \binom{m}{1}\binom{n-1}{0} + \binom{m}{2}\binom{n-1}{1} + \cdots + \binom{m}{r}\binom{n-1}{r-1} + \cdots + \binom{m}{m}\binom{n-1}{m-1}$$

   In sum notation that looks like this:

$$\binom{n+m-1}{n} = \sum_{r=1}^{m} \binom{m}{r}\binom{n-1}{r-1}$$