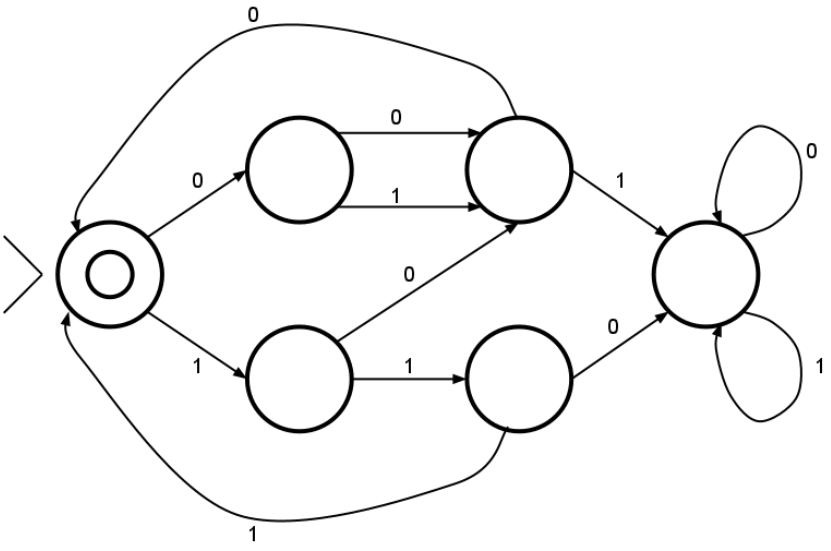
Math Circle - Winter 2012 - Homework 1

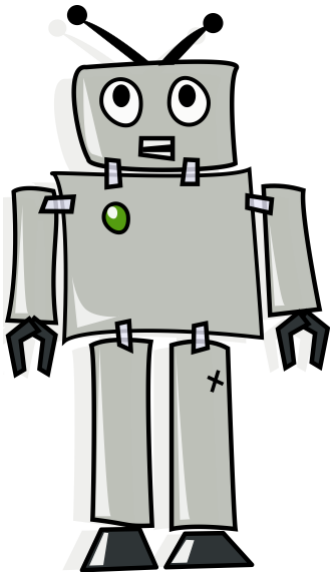
1. (10 points) Daniel is an automaton which is terrified of long strings of the same letter. Daniel takes as input any string consisting of a and b. He will REJECT a string if it ever has at least three a in a row, or at least three b in a row. Otherwise, he will ACCEPT it as a perfectly reasonable input. Draw Daniel's states and arrows.

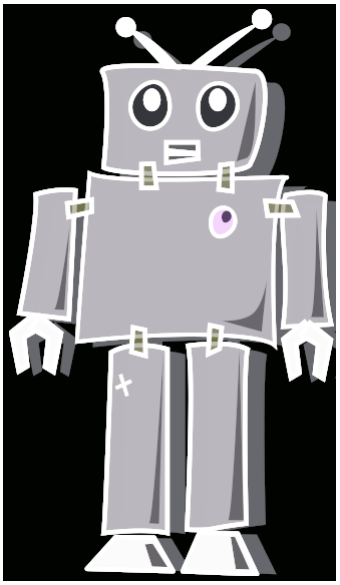
Examples: ACCEPT - *aabbaabb*, *abbaababba*, *aab*;
REJECT - *abaaabbbaba*, *bbbbbbb*, *baaaab*.

2. (10 points) Lisa is the complicated-looking automaton pictured below. She has an extremely important job in the inner-workings of much more complicated machines, because there is a very nice and useful mathematical explanation for the strings which Lisa ACCEPTs. Describe as simply as possible all the strings of 0 and 1 which are ACCEPTed by Lisa. Why would Lisa be so useful?



3. (10 points) Andy is an automaton. We wish to construct Andy's *perfect match* automaton, call it Brian. Brian should ACCEPT exactly those inputs which Andy REJECTs, and also REJECT exactly those inputs which Andy ACCEPTs. Explain how to construct Brian, assuming we know the inner-workings of Andy.





4. (10 points) Do you think it is possible to construct an automaton which ACCEPTs only those strings of a and b of the form $a^n b^n$ for some nonnegative integer n? If possible, draw the automaton. If you do not think it is possible, give a convincing argument.

Examples: ACCEPT - *ab*, *aabb*, *aaabbb*, *aaaabbbb*;
REJECT - *aa*, *aaabb*, *ababab*, *aabbaaabbb*.