Math Circle - Winter 2012 - Homework 2

1. (10 points) The game Yahtzee involves rolling 5 six-sided dice at once. How many different outcomes of such a roll are possible?

2. (5+10 points)

(a) In how many ways can you place 5 Xs and 4 Os on a Tic-Tac-Toe board?

(b) What is the total number of boards from part (a) in which there is at least one row, column, or diagonal filled with three Os?

3. (10 points) 10 men named Jon, 8 named John, and 7 named Johan entered a race. In how many distinguishable ways can these 25 men finish the race? Two finishes are considered *indistinguishable* if the order of names is the same in the final ranking.

4. (5+5 points) Recall that $n! = (n)(n-1)(n-2)(n-3)\cdots(3)(2)(1)$.

(a) Let p be a prime number. Prove that $(p-1)!$ is not divisible by p.

(b) Let n and k be positive integers. Under what conditions is $\dfrac{n!}{k!}$ also an integer?