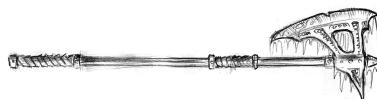


## Math Circle - Winter 2012 - Homework 4

1. (10 points) Ents are very old forest creatures. Suppose that 15 Ents have been standing in the same short row for a very, very long time. After an unbelievably long conversation in Entish, the Ents decide that some number of them (possibly zero) will break out of the row, take one large step forward, and start a new row. One condition: they are bored of the same Ents around them. So if two Ents are standing next to each other, a maximum of one of them is allowed to move forward (that is, they cannot **both** move forward). In how many different arrangements can the Ents end up after doing this seemingly trivial task?



2. (10 points) The hobbits of the Shire have created a long piece of chocolate consisting of 20 squares in a line. Hobbits consider it bad luck to have an even number of chocolate squares, so the chocolate bar is to be broken up from left to right into smaller pieces consisting of only odd numbers of chocolate squares each. In how many different ways can they accomplish this? *Note.* Each square is slightly different-tasting, so the order of breaking matters. For example, breaking off 1 square followed by 3 is different than breaking off 3 squares followed by 1!



3. (10 points) The dwarves of Middle-earth enjoy mining above all else. When swinging a pickaxe, a dwarf makes exactly 20 swings at a time: 10 to the left and 10 to the right. The dwarves have a single point of superstition. They believe that if at any given point of time in a 20-swing procedure, a dwarf has made more rightwards swings than leftwards swings, then a demonic Balrog is awakened to terrorize the mines. How many different 20-swing patterns which will not awaken a Balrog are available to the dwarves?



4. (10 points) Suppose that 8 wise elves are attending a banquet in their honor at the House of Elrond. Upon entering, each elf hands his or her cloak to the hobbit hired to keep track of them for the night. After the banquet, the confused hobbit returns each cloak in such a way that not a single elf has been given his or her original cloak. In how many ways can the hobbit accomplish such a mix-up?