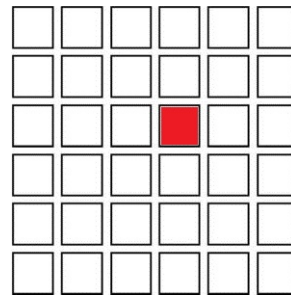
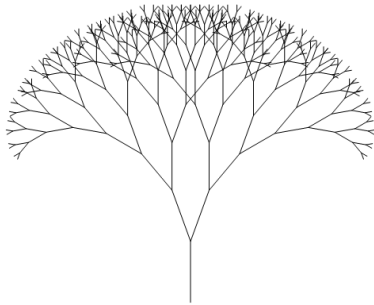


Math Circle - Winter 2012 - Homework 5

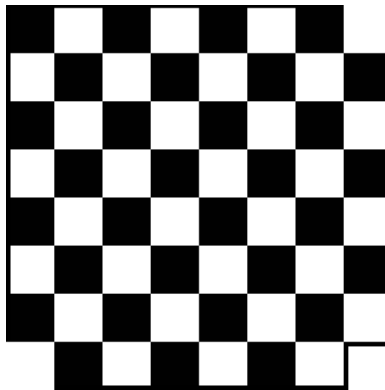
1. (10 points) A mathematician is wandering through the desert when he comes across a tree with 25 apples and 30 oranges. This tree is magical: when you pick two fruit of the same kind from the tree, another orange grows back; when you pick two different kinds of fruit, an apple grows back. The mathematician was very hungry and ate until there was only one fruit left hanging — the final fruit which grew back after taking the tree's last two fruits. Was this final fruit an apple or an orange?



2. (10 points) Four of the Northwest's sneakiest thieves are standing on a 6×6 square grid, one on each corner. The thieves have received a credible tip that the Queen's royal comic book collection is hidden on one of the squares near the center of the grid (specifically the square in the 3rd row, 4th column). The thieves begin walking around the grid, moving to an adjacent square exactly at the start of every minute. Adjacent squares are north, south, east, and west (if available); but **not** diagonal. They must move like this in order to thwart the security system of the Queen's grid. A thief is not allowed to stay in the same square as a move.

Each thief has a specific skill which is required to crack the comic book lock, and all the thieves must act together. Can you give a sequence of legal moves for the thieves so that at some point they all end up in the comic book square at the same time? If you cannot, explain why you do not think it is possible.

3. (10 points) A knight (horse) is sitting in the NW corner of an 8×8 chessboard which has had its other three corners removed. Is it possible for the knight to move around this board, touching each square exactly once? Recall that knights can only move in L-shapes.



4. (10 points) *Pika Chu's* is a certain Pokémon trading post which trades only three types of Pokémon: fire, water, and grass. Trades are made based on Pokémon level and fighting moves, but here they have one hard-and-fast rule. If you give *Pika Chu's* one of any of the three acceptable types, then you must take exactly one each of the other two types.

Suppose *Pika Chu's* starts the day with 13 fire, 5 water, and 12 grass type Pokémon. In the late afternoon they close up with a single Pokémon on their shelf. What is the type of that Pokémon?

You give	You take
fire	water + grass
water	fire + grass
grass	fire + water