Math Circle - Homework 6

1. (10 points) Start with the numbers

$$-5, -4, -3, \ldots, 0, \ldots, \ 3, \ 4, \ 5$$

written on the board. You are permitted to come up and erase any two numbers a and b, and then write the new number $\sqrt{a^2 + b^2}$. Perform that action exactly ten times, and you'll end up with a single number on the board. Can you determine what this final number will be? More importantly, why is this final number always the same?

2. (10 points) There are 7 obscrure clocks on the wall of a mathematician's office. Each clock has no hour-hand, and all the minute-hands are pointing straight up. In one move, you are allowed to move the minute hands of any four clocks up by exactly 15 minutes. Is it possible to reach a situation in which all the minute-hands are pointing straight down?

3. (10 points) Prove that a 10×10 board cannot be covered without overlapping by tiles of the shape shown below: