MATH CIRCLE - INTUITION WITH SETS

In this worksheet, A, B, and C all denote sets.

(1) For each of the following, either give an example or explain why such an example is impossible.

    (a) Two finite sets A and B such that $A \cup B$ is infinite.

    (b) Two infinite sets A and B such that $A \cap B$ is finite.

    (c) An infinite set A and an infinite subset $B \subseteq A$ such that the set difference $A - B$ is finite.

    (d) An infinite set A and an infinite subset $B \subseteq A$ such that the set difference $A - B$ is infinite.

    (e) An infinite set A and a finite subset $B \subseteq A$ such that the set difference $A - B$ is also finite.

    (f) Two different sets $A \neq B$ such that $A - B = \emptyset$.

(g) A finite set A such that the power set $P(A)$ has exactly 5 elements.

(h) An infinite set A and two infinite subsets $B, C \subseteq A$ such that both $B \cap C = \emptyset$ and $B \cup C = A$.

(2) What can you say about the following sets?

(a) $A \cup A$

(b) $B \cap B$

(c) $(A \cap B) \cap C$

(d) $(A \cup B) \cap A$

(e) $(A \cup B) \cap C$

(f) $(A \cap B) \cup C$

(3) If $|A| = n$, what is the size of its power set $P(A)$?

(4) Write a formula for $|A \cup B|$ in terms of $|A|$, $|B|$, and $|A \cap B|$.