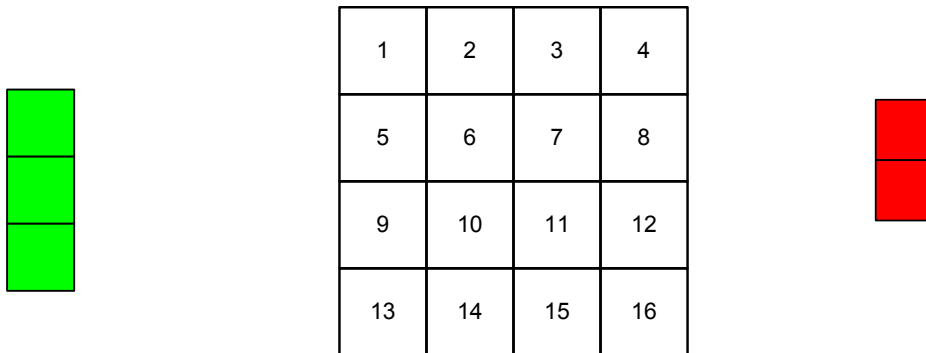


Math Circle - Battleship

In a simplified version of Battleship, your team places its two ships on a 4×4 grid, either horizontally or vertically, with no overlap. One of your ships has a length of 2 units, and one has a length of 3 units.



The grid is numbered 1 – 16. In order to keep your positions more secret from prying eyes, your team decides to multiply each of the board positions by a certain number N , then record the positions modulo 17. N is not allowed to be a multiple of 17.

For example, suppose your team has a ship at positions 7 and 11, and that you are using $N = 3$ as an encoding number. We know that

$$7 \cdot 3 = 21 \equiv \mathbf{4} \pmod{17}$$

$$7 \cdot 11 = 77 \equiv \mathbf{9} \pmod{17}.$$

You would record ships at positions 4 and 9. Notice that your recorded positions are not even necessarily next to each other anymore!

1. An enemy team reports the following positions for locations of their two ships. Your spies are able to confirm that the number they used to encode their positions is $N = 5$. What is the original configuration for their ships?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

2. Spy A obtains an encoded partial map for the enemy which says that there is a ship at position 4, but no ship at position 3, 7, nor 8.

Spy B informs you that he can confirm that the same enemy is using the number $N = 4$ for encoding.

How can you convince your superiors that at least one of your spies is lying – or at least has faulty information?

3	4
7	8

3. You and your team should create a secret configuration of your two ships. One of your ships **MUST** start at the square labeled 1. Now, pick a secret N and use it to encode your ship positions as described above. Trade ENCODED positions with another team, and try to figure out their true ship layout.

4. Another enemy team wishes to play the same game on a 5×5 grid labelled 1 – 25. When encoding, what number would you use to reduce (like 17 above)? Would you need to place more restrictions on the encoding number N ?