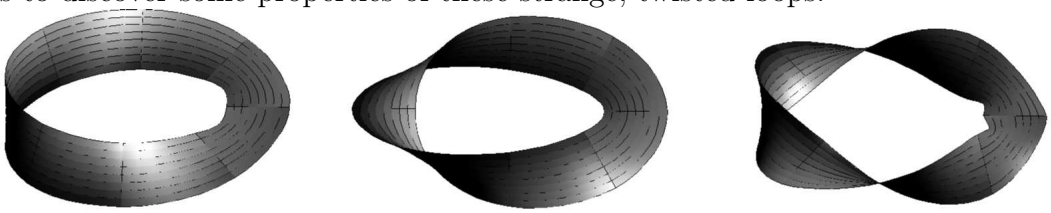
Math Circle - Möbius Fun

Q: *Why did the chicken cross the Möbius strip?*
A: *To get to the same side.*

Your group should have received four long strips of paper: three blank ones and one with stripes drawn. Fold each strip into loops, but, just before taping the ends together, give it a twist (or two, or three):

Groups 1 and 2. 1 twist;
Group 3. 2 twists;
Group 4. 3 twists.

The goal is to discover some properties of these strange, twisted loops.



1. Draw a dot somewhere in the middle of one of the blank strips. Trace a path around the strip. How many times do you go around before you come back? From this, conclude how many sides the loop has.

2. Put a dot on the edge of the strip. Trace your finger along the edge. How many edges does the loop have?

3. Take another one of the blank strips and cut it along the middle (try to stay in the middle!) until you come back where you started. What do you end up with? How many sides and edges does the new surface have?

4. Now cut again along the middle of the new surface. What do you get this time?

5. Take the loop that has lines drawn on it and cut along the pencil lines. Do you have to make two cuts or just one? What kind of surface(s) do you get?

6. Take two strips and tape their edges together to make a wider strip. Now try to keep gluing them together along their edges. Can you succeed?