Math Circle - Countin' Pirates

YARR! Today we be shippin' out on yer maiden voyage o' me old favorite pirate ship, the Katy Lan. You best not be skirtin' yer duties matey – thar be a special place on th' plank fer those among ye who cannot be bothered to finish yer tasks!

1. We be bound to come across some shady ninjas on our journey thru the sea. Each cannon needs to have a fresh stack of cannonballs within quick grasp fer just such the occasion! The cannonballs are most accessible if they are stacked right on top o' each other, directly against the wall of the Katy Lan.

   But we be pirates, not savages! In order to keep the ship lookin' perty, I proclaim that no two stacks of cannonballs are allowed to look the exact same! How many ways can ye stack the cannonballs if there be a full row of 4 cannonballs at the bottom of the stack? What about startin' with 5 cannonballs? 10?

2. We better be takin' some ninja prisoners after any raids on our ship! And we better be takin' these prisoners in pairs! Each time we take a pair of prisoners on board the Katy Lan, those two ninjas get chained together for the rest of the voyage. And as you all know, I require my prisoners to sing to me while I eat my dinner!

   At every dinner, ye need to bring up those no-good ninjas to me quarters and stand them in a line directly in front of me table, so I can hear thar squalid ninja songs. But I don't like no chains clanking while I enjoy me dinner, so ye best arrange the pairs of prisoners so that none of their chains overlap. How many ways can you arrange the pairs in a line if there are 4 pairs of prisoners? What about 5 pairs? 10?

   In this problem, a ninja is a ninja is a ninja – all ninjas be considered the same. What we really wanna know is how many different ways there be of arrangin' the chains.

3. My favorite piece of booty from any pirate raid be a fancy manilla envelope! ARRR – thar be a place on th' plank fer those of ye laughin' at my envelopes, too! I keep my extensive collection stacked upright in me personal treasure chest. I trust you to keep me entertained by making sure that the envelopes are always in a different arrangement each time I open me chest!

   Ye be encouraged to stack the envelopes in any way ye see fit, even possibly nested inside of each other. How many ways can you arrange me treasure chest if I have 4 manilla envelopes? What about 5 envelopes? 10?

4. We be needin' some target practice for them high-sea encounters with the ninjas. An' despite our lazy-lookin' demeanor, we actually be quite accurate with our pistols. To make sure we don't be gettin' too full of ourselves, we need to be constantly practicin' on diff'ernt targets.

   On the Katy Lan, all our pistol targets be strings of circles. By that I mean that we draw a bunch of circles (of possibly different sizes) with all their centers lyin' on a single line. These circles shouldn't intersect each other, but they can lie nested inside of each other. How many different arrangements of 4 circles can you design fer us as targets? What about with 5 circles? 10?

5. We pirates not be all work an' no play. Ay, we love playin' Treasure Hunt! I will set up a square grid which is one step wide fer each square, and you start on the SW corner. The treasure be buried at the NE corner, and it be yer task to walk from where ye be to where the treasure be.

   Thar be a few rules. First, from where ye start, look directly NE towards the treasure. Any corners on the grid which lie to yer left have been poisoned, so ye don't wanna be walkin' that-a-way. Second, ye be only allowed to take one step at a time directly to the east or one directly to the north. Each step you must move from one corner of the grid to an adjacent corner. How many different paths be there fer you to get to the treasure if you start on a 4×4 grid? What about a 5×5 grid? 10×10?