# Math Circle - Combinatorics

**Combinatorics** is the fancy word that mathematicians say when they mean *counting*. There are quite a few combinatorial techniques. The multiplication and addition principles are two. We also slightly touched on something called the *inclusion-exclusion principle* when we were first learning sets. We will learn many, many more ways to count!

Do the following exercises. In each exercise, keep track of where you use the multiplication and addition principles.


1. A decimal integer is called *odd-looking* if all of its digits are odd. How many odd-looking 5-digit numbers are there?

2. In how many different ways can you form a subset of the set $\{1, 2, 3, 4, 5\}$?

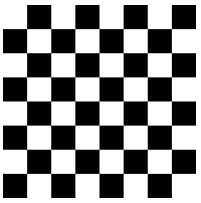
3. There is an extremely small island near the middle of the Pacific Ocean on which the inhabitants have only four letters in their written language: $\forall$, $\exists$, $\natural$, and $\otimes$. Also, every combination of letters is a word in their language. How many words do the islanders have consisting of no more than 5 letters?

4. In how many different ways can you form a subset of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ which contains at least one prime number?

5. Chris and Kolya want to trade Pokémon. Chris has 7 fire-type Pokémon, 5 water-type, and 4 grass-type. Kolya has 3 fire, 5 water, and 6 grass. They are willing to trade only one Pokémon each, and only for the same type. In how many different ways can they accomplish this?

6. In a high school election there are 10 candidates running for office. The person who receives the most votes will be the president; the candidate with the second-most votes will be vice president; and whoever gets third-most votes is elected as the vice vice president. How many ways are there of electing the president, vice president, and vice vice president?

7. How many ways are there of placing one black rook and one white rook on an $8 \times 8$ chessboard so that they cannot attack each other?



8. How many ways are there of placing one black king and one white king on an $8 \times 8$ chessboard so that they cannot attack each other?

9. Bethany has 7 different socks and wishes to arrange them in a row. In how many ways can she do this? In general, how many ways are there to arrange $n$ different objects in a row? *Hint.* Recall the factorial notation ($n!$).

10. There are 7 delicious pies cooling on a windowsill, accompanied by 4 ravens closely eyeing them. In how many ways can each raven eat exactly one pie?

11. How many different ways can you rearrange the letters in the following words:

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12. How many diagonals are there in a convex $n$-gon?

13. King Arthur has invited all of his knights to a feast. How many different ways are there of sitting all 13 knights at their Round Table? Two configurations which are rotations of each other are considered to be equal.