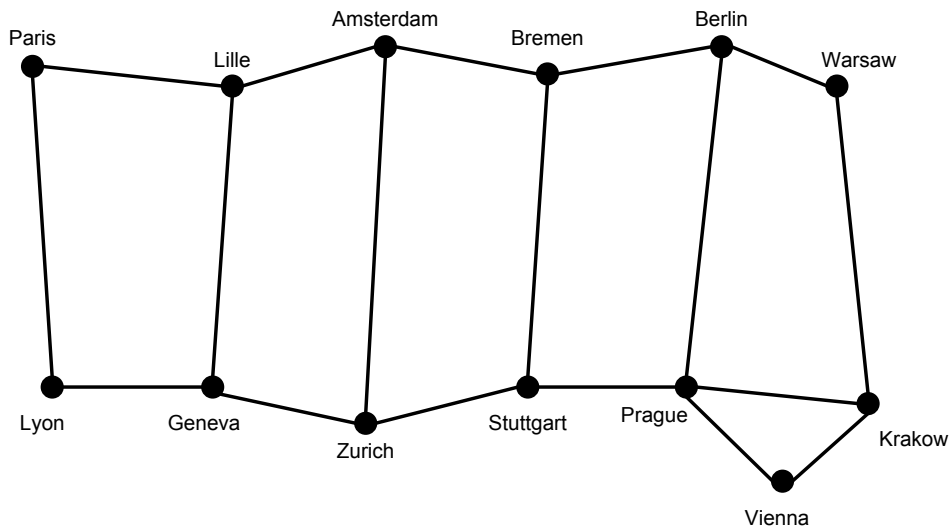


# Math Circle - Some Games

1. Newton and Leibniz are playing a game with the following rules.
  - (a) Newton begins by placing a knight on any square of an  $8 \times 8$  chessboard.
  - (b) Leibniz moves the knight first.
  - (c) Newton and Leibniz alternate moving the knight, but they can only move the knight to squares it has not yet visited.
  - (d) The player who is unable to move the knight loses.

Can you help either Newton or Leibniz win the game, no matter what the other player does?

2. Newton has stolen Leibniz's collection of large curly wigs and hid away in the central Europe. Leibniz follows him, but misses the correct city by a bit — he will need to travel. Because of the way European roads are constructed, the two mathematicians can only travel by certain roads, as indicated in the map below. Newton starts in Paris, and Leibniz in Geneva. They take turns moving from city to city, and if Leibniz ever moves into the same city as Newton, he will be able to get his wigs back. Leibniz moves first. Will he be able to recover his wigs?



3. In retaliation for the wig-snatching, Leibniz has set up a game for Newton. He has placed 4 cards labeled 1, 2, 3, and 4 in a row on the table, in increasing order 1234. As a move, Newton is allowed to switch any two adjacent cards. Newton wins if he can reverse the order of the cards (to 4321) in an *odd* number of moves.
- (a) Can Newton ever win this game?
  - (b) If not, can Newton at least reverse the order in an *even* number of moves?
  - (c) What if Newton is allowed to switch *any* two distinct cards (not necessarily adjacent) as a move?

