# Math Circle - Multiplication and Addition Principles

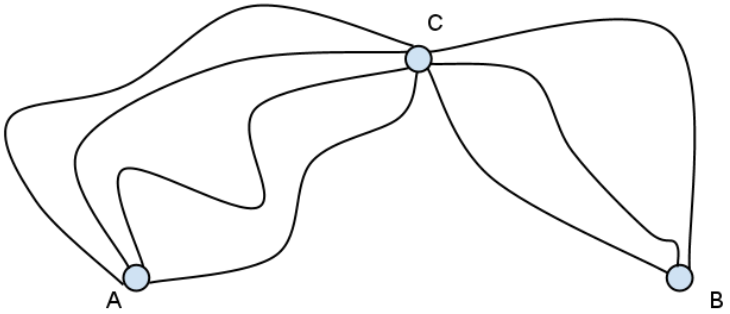
1. Today is Mother's Day. Chris wants to make his mom a card by gluing one large heart onto a piece of construction paper. There are 5 different colors of construction paper, and 3 different types of hearts. How many different combinations of paper and hearts are available to Chris for making the card?

2. Chris also decides to decorate the border of the card. There are 7 different borders available to him. Now how many different cards can he make?

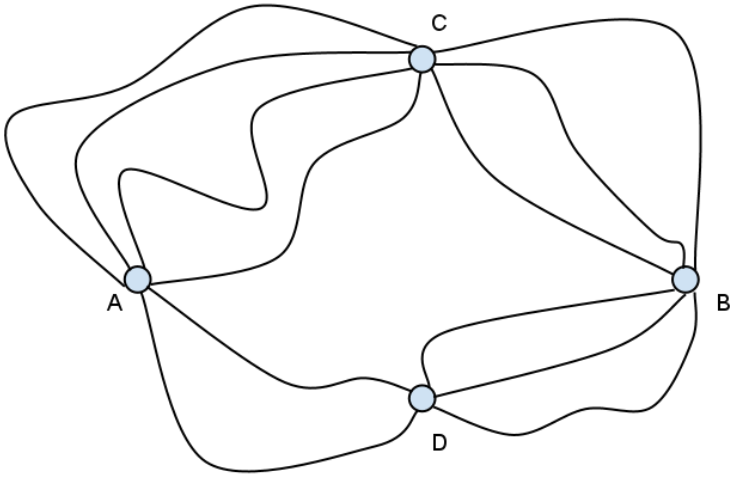
3. At a simplified Chipotle, you can order a burrito with exactly one choice from each of the following options. How many different burritos are possible?

   - White or brown rice.
   - Black beans, pinto beans, or no beans.
   - Chicken, steak, barbacoa, carnitas, or no meat.
   - Mild, medium, hot, or no salsa.
   - Cheese or not.

4. In the following map, towns A and B are connected to each other by roads, all via town C. How many different paths can you take from A to B?



5. After few months, a new town D has been added to the map above, with roads as shown. Now how many different paths can you take from A to B?




6. Kolya is at the pet store deciding between either a frog or a hamster. If he gets a frog, Kolya has a choice between 4 species and 6 different aquariums. If he gets a hamster, Kolya can choose one of 7 colors and one of 5 cages. How many options does Kolya have for purchasing his first pet?

The above problems illustrate the following two counting principles, which serve as our basis for **combinatorics**. In both statements, *Event 1* and *Event 2* are two events whose occurrences are independent (i.e. one does not affect the other).

**Multiplication Principle.** If there are $m$ ways in which *Event 1* can occur and $n$ ways in which *Event 2* can occur, then there are $m \cdot n$ ways in which both *Event 1* and *Event 2* can occur together.

**Addition Principle.** If there are $m$ ways in which *Event 1* can occur and $n$ ways in which *Event 2* can occur, and it is not possible for *both* events to occur, then there are $m + n$ ways in which either *Event 1* or *Event 2* can occur.