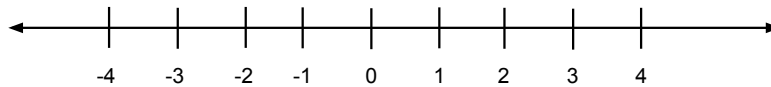


Math Circle - Puzzles

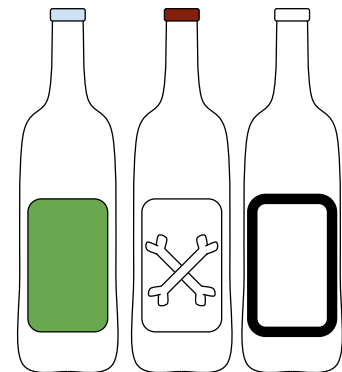
1. A spy is located on an infinite number line.



At time 0, the spy is at location A . With each time interval, the spy moves B units to the right (if B is negative, the spy is moving left). A and B are fixed integers, but they are unknown to you. You are to catch the spy. The means by which you can attempt to do this is: at each time interval (starting at time 0), you can choose a location on the line and ask whether or not the spy is currently at that location. That is, you will ask a question like “Is the spy currently at location 27?” and you will get a yes/no answer. Devise an algorithm that will eventually find the spy.

2. Someone hands you an unfair 2-sided coin. How can you get a fair $1/2$ chance using only this coin?

3. An evil king has a cellar of 1000 bottles of very expensive wine. A neighboring queen plots to kill the bad king and sends a servant to poison the wine. (Un)fortunately the king’s guards catch the servant after he has only poisoned one bottle. Alas, the guards do not know which bottle but know that the poison is so strong that even if diluted 1,000,000 times it would still kill the king. Furthermore, it takes one month to have an effect.



The evil king decides he will get some of the prisoners in his vast dungeons to drink the wine to check if it is poisoned. He’ll be able to spin a low prisoner death rate — what is the smallest number of prisoners the king needs, in order to be able to drink all the un-poisoned wine at his anniversary party in five weeks time?