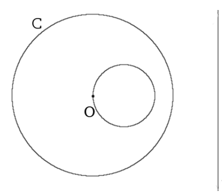
UW Math Circle - Homework 8

*Recall that an **inversion** with respect to a circle C centered at O with radius r is a map that takes any point P to a point P' that lies on the ray OP such that*

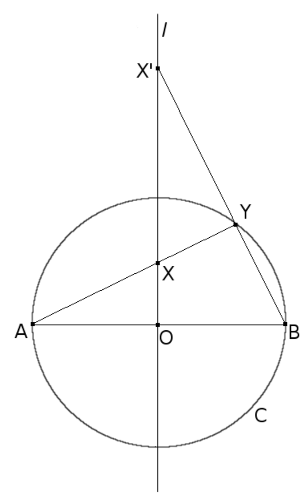
$$|OP| \cdot |OP'| = r^2$$

1. Suppose the point $(0,0)$ is the center of a circle C of radius 4. What are the inverses of the points $(1,0)$, $(1,2)$, $(3,0)$, and $(3,2)$? If I connect these four points with line segments, what do the inverses of the line segments look like?



2. In class we showed that inversions with respect to circles of radius 1 take lines to circles that go through the origin. Now suppose O is the center of a circle C with radius r. Prove that an inversion with respect to C takes a line that does not pass through the origin to a circle that does pass through the origin. Find the radius of the circle.

3. Let C be a circle with center O and diameter AB. Let ℓ be the line perpendicular to AB through O. Now let X be any point on ℓ different from O and let Y be the second intersection of the line AX with C. Finally, let X' be the intersection of BY with ℓ. Show that X' is the inverse of X.



4. 100 cars are parked along a road, of which 30 are red Ferraris, 20 are yellow Ferraris, and 20 are black Ferraris. No two Ferraris of different color are standing next to each other. Prove that there are three Ferraris standing in a row that are the same color.