

Problem Set 9

UW Math Circle – Advanced Group

Session 14 (23 January 2014)

1. Write down your solution to #1 of last week's homework.
2. (a) Does the set of even integers with the operation of addition form a group? What about multiplication?
(b) Same questions for the set of real numbers.
(c) Same questions for the set of positive real numbers.
3. Let G be a group and suppose that for all $a, b \in G$, $(a * b)^{-1} = a^{-1} * b^{-1}$. Prove that G is abelian.
4. Let G be a group and $a, b \in G$. Show that if $(a * b)^n = 1$, then $(b * a)^n = 1$.
5. (a) Let G be a finite group and $a \in G$. Show that there exists a positive integer n such that $a^n = 1$. The smallest such n is called the *order* of a .
(b) A sequence of operations was applied to a Rubik's Cube. Show that by repeating this sequence of operations sufficiently many times the cube can be returned to its initial state.

