Problem Set 18

UW Math Circle – Advanced Group

Session 25 (1 May 2014)

1. Use the Robinson-Schensted correspondence to prove the *Erdös-Szekeres theorem*: Given any permutation of $1, 2, \ldots, mn+1$, the permutation contains either an increasing subsequence of length $n+1$ or a decreasing subsequence of length $m+1$. (For example, for $m = n = 2$, any permutation of $1, 2, 3, 4, 5$ has either an increasing or decreasing subsequence of length 3.)

2. (Erdös-Szekeres game) A and B play a game. The first player writes a digit (1 to 9) on the board, then the second player appends on the right a digit that has not yet been used, etc. A player loses if he or she creates a sequence that has either an increasing subsequence of length 4 or a decreasing subsequence of length 3.

   (a) By the Erdös-Szekeres theorem there will be a loser by the time all digits from 1 to 9 are placed.

   (b) Who has a winning strategy?

   (If the 3 in "decreasing subsequence of length 3" is replaced by any larger number, the winning strategy is unknown!)

3. (MHO 2010) Alex, Bob and Chad are playing a table tennis tournament. During each game, two boys are playing each other and one is resting. In the next game, the boy who lost rests, and the boy who was resting plays the winner. By the end of tournament, Alex played a total of 10 games, Bob played 15 games, and Chad played 17 games. Who lost the second game?