

Problem Set 2

UW Math Circle – Advanced Group

Session 3 (10 October 2013)

Recall the definitions we made about a set S in the plane:

- A point x is an *interior point* of S if there is a small disc around x that is entirely contained in S .
- A point x is an *exterior point* of S if there is a small disc around x that is entirely outside of S .
- A point x is a *boundary point* of S if it is neither an interior point, nor an exterior point.
- S is *open* if it does not contain any of its boundary points, *closed* if it contains all of its boundary points.

1. Show that if A and B are open sets, so are $A \cup B$ and $A \cap B$.
2. Which sets in the plane are both open and closed? (Hint: There are two of them.)
3. Let S be the set of points in the plane with coordinates (x, y) , where x and y are rational numbers. Find the boundary and interior of S .
4. (Russian Mathematical Olympiad, 2012) 101 wise men are standing in a circle. Each of them thinks either that Earth is flat, or that Earth has the shape of a torus. Once a minute, the wise men all state their opinions on the shape of the Earth. Immediately after this, each wise man both of whose neighbors disagree with him changes his opinion. Prove that after some time the wise men will stop changing their views on this contentious issue.

