Problem Set 3

UW Math Circle – Advanced Group

Session 4 (17 October 2013)

1. Suppose that S is a convex set in the plane and that x and y are points in the interior of S. Show that all points on the segment connecting x and y are also in the interior of S.

2. (Helly's theorem in one dimension) The United Galactic Senate is debating next year's budget. Each of the 100 Senators has minimum and maximum budgets that he or she is willing to tolerate (for example, no less than \$100 trillion, but no more than \$150 trillion).

   It turns out that any pair of Senators can agree on a budget. Prove that all 100 Senators can agree on a budget.

3. (Radon's theorem in two dimensions) Four points are given in the plane. Prove that they can be split into two subsets whose convex hulls intersect.

4. During a long budget debate in the United Galactic Senate, each Senator threw a space rock at exactly one other Senator. Prove that they can split into three parties in such a way that no one has thrown a space rock at anyone in his or her own party.