# Problem Set 4

## UW Math Circle – Advanced Group

## Session 5 (24 October 2013)

1. (Russia 1997) For which $\alpha$ does there exist a nonconstant function $f : \mathbb{R} \to \mathbb{R}$ such that $f(a(x + y)) = f(x) + f(y)$ for all $x$ and $y$?

2. 2014 numbers are written in a row. It is known that each number is the sum of its two neighbors. If the first number is 1; what could the last number be?

3. (Cauchy's functional equation) Suppose $f : \mathbb{Q} \to \mathbb{Q}$ satisfies $f(x + y) = f(x) + f(y)$ for all $x$ and $y$. Prove that $f(x) = ax$ for some $a \in \mathbb{Q}$.

4. In a certain herd of 33 cows, each cow weighs a whole number of pounds. Farmer Dan notices that if he removes any one of the cows from the herd, it is possible to split the remaining 32 cows into two groups of equal weight, 16 cows in each group. Show that all 33 cows must have the same weight.