UW Math Circle
October 17, 2013

1. What is the last digit of the sum $1^2 + 2^2 + ... + 99^2$?

2. How many ways are there to place a white rook and a black rook on a chessboard in a way that they cannot attack one another?

3. Jake and Sarah are trying to divide their cousins into teams. They have 2 girl cousins and 4 boy cousins.

   (a) How many ways can they make two teams of 3?

   (b) If each team must have at least one boy and one girl, how many ways can they make two teams of 3?

   (c) How many ways can they make 3 teams of 2?

   (d) How many ways can they divide their cousins into two groups (not necessarily of the same size)?

4. A mail carrier delivers mail to the nineteen houses on the east side of Elm Street. The carrier notices that no two adjacent houses ever get mail on the same day, but that there are never more than two houses in a row that get no mail on the same day. How many different patterns of mail delivery are possible?

5. How many 3 digit numbers have no two adjacent digits the same? (For example, 124 is an acceptable number but 566 is not.) How about 4 digit numbers? 5 digit numbers?

6. Is it possible for 2^n to have the same digits as 2^m (in base 10), just rearranged, for $n \neq m$ natural numbers?