UW Math Circle January 9, 2014 Homework

The Pigeonhole Principle: If N + 1 pigeons fly into N birdhouses, then some birdhouse will contain at least 2 pigeons.

The Generalized Pigeonhole Principle: If $k \cdot N + 1$ pigeons fly into N birdhouses, then some birdhouse will contain at least k + 1 pigeons.

1. No person has more than 500,000 hairs on his/her head. Show that two people in New York City have the same number of hairs on their head.

2. Show that if you place 41 rooks on a 10×10 chessboard, then there will be at least five of them that do not attack one another.

3. If you write the numbers 1, 2, ..., 10 around a circle (in any order), show that there must be 3 adjacent numbers whose sum is greater than or equal to 17.

4. Donald has a new business suit with 10 pockets in it, and he wants to put 44 business cards into his pockets. Can he do this in such a way that each pocket has a different number of cards?