UW Math Circle<br>January 9, 2014<br>Homework

The Pigeonhole Principle: If $N+1$ pigeons fly into $N$ birdhouses, then some birdhouse will contain at least 2 pigeons.

The Generalized Pigeonhole Principle: If $k \cdot N+1$ pigeons fly into $N$ birdhouses, then some birdhouse will contain at least $k+1$ pigeons.

1. No person has more than 500,000 hairs on his/her head. Show that two people in New York City have the same number of hairs on their head.
2. Show that if you place 41 rooks on a $10 \times 10$ chessboard, then there will be at least five of them that do not attack one another.
3. If you write the numbers $1,2, \ldots, 10$ around a circle (in any order), show that there must be 3 adjacent numbers whose sum is greater than or equal to 17 .
4. Donald has a new business suit with 10 pockets in it, and he wants to put 44 business cards into his pockets. Can he do this in such a way that each pocket has a different number of cards?
