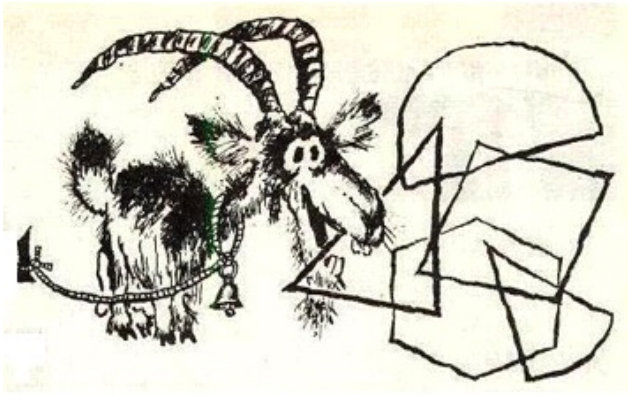
Goats on a string and the UW Math Circle

March 6, 2014



Today's problem set deals with goats. Goats are ravenous and consume everything they can reach. Because of this, they are usually kept on a rope.

1. Draw the section of a pasture consumed by a goat if the goat is tied to a single stake planted in the pasture.
2. A mathematician took a walk on a field holding a goat on a 1-meter-long rope. The mathematician's path was rectangular with dimensions 3 meters by 5 meters. Draw the section of the field the goat will have consumed by the end of the mathematician's walk.
3. How can a goat be constrained to an eye-shaped field? In other words, how can the goat be tied using ropes and stakes, so that it can eat only the grass in the field?
4. A rope has been stretched between two stakes in a field. A goat is tied to this rope with another rope that is free to slide along the first rope. What is the shape of the portion of the field the goat can eat?
5. How can a goat be constrained to a field in the shape of a
   (a) semicircle?
   (b) square?
   (c) rectangle?
6. How can a goat be constrained to a field in the shape of a
   (a) triangle?
   (b) regular hexagon?
7. Dogs can be used to herd goats because a goat will not occupy a space that a dog can reach. However don't let a dog run free since it will chase the goat constantly, and never let the goat rest or eat.
   (a) How can one dog hold a goat in a ring?
   (b) What about in a semicircle?
   (c) Using dogs, contain an untied goat in a triangle.
8. (a) The fence of the Goats-R-Us farm is triangular in shape. Two goats are tied to the fence with ropes, each at the midpoint of two different sides. The lengths of the ropes are equal to half the lengths of the section of fence they are tied to. Can the goats eat all of the grass inside the fense?
   (b) What if the fence is a quadrilateral, and there are goats tied at the midpoints of each of the four sides with ropes that are half the length of the side they are tied to?

Extra problems

9. In how many different ways can you fill in the blanks with standard operations (plus, minus, times, division, exponent) and place parentheses as needed to make true statements?

$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 1$$
$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 2$$
$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 3$$
$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 4$$
$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 6$$
$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 7$$
$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 8$$
$$1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5 \_\_ 6 \_\_ 7 \_\_ 8 \_\_ 9 = 9$$

10. In how many ways can you fill an $m \times n$ checkerboard with 0's and 1's so that the sum of the numbers in each row and each column is even?