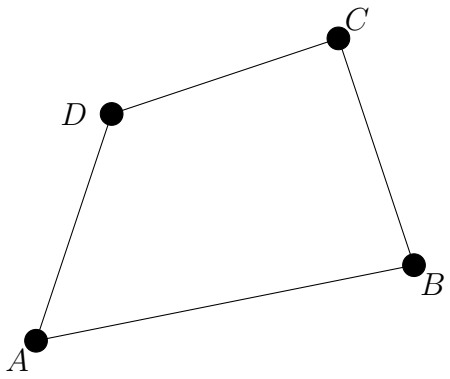
UW Math Circle

March 13, 2014

1. Let $ABCD$ be a convex quadrilateral. Find the point P in the interior of the quadrilateral that minimizes the sum of the distances

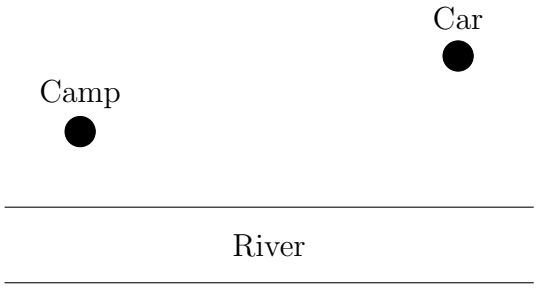
$$|AP| + |BP| + |CP| + |DP|.$$



2. There are 100 delegates from Mars and Venus attending the Intergallactic Conference of Mathematics. During the conference, all of the delegates sit around a large circular table. If there are 55 Martians at the conference, prove that two of them must be sitting directly across from one another.

3. Let's say that a *lattice point* on a piece of graph paper is a point where two grid lines meet. Prove that if you choose any five lattice points, then there will be two of the points so that the midpoint of the line between them is also a lattice point.

4. Camper Carl is spending the weekend at Olympic National Park. He has set up his tent, and has one more load of gear to trek from his car to the campsite. In order to save time, he will stop at the river to gather water on his way back to the campsite. Where should he stop along the river in order to minimize the total distance he has to walk?



5. The distance from Seattle to Los Angeles is 1350 miles. The distance from Los Angeles to Reno is 630 miles. The distance from Reno to Bend is 375 miles. The distance from Seattle to Bend is 345 miles. How far is it from Bend to Los Angeles?

6. You are given a 4×4 grid of numbers. It is allowed to either increase all of the numbers in a row by 1 or decrease all of the numbers in a column by 1. Is it possible to turn the table on the left into the table on the right using these operations?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	15

7. An ant sits at one corner of a wooden block. What is the shortest distance it can take to walk to the opposite corner of the block?

8. Two cities are located on opposite sides of a river. The cities have agreed to build a bridge across the river. If the bridge has to be perpendicular to the river, where should they build the bridge so that the total distance from the first city to the bridge, across the river, and directly to the second city is as short as possible?