Problem Set 5

UW Math Circle

Session $\omega + 7$ (6 November 2014)

1. (Russia 2012) 73 teams participated in a very long frisbee tournament in which every team played each other team once. After the tournament, the teams were split into two groups such that every team in the first group won m games and every team in the second group won n games. Is is possible that $m \neq n$?

   (Note that there are no ties in frisbee.)

2. (Moscow 1999) At a chess tournament, each player played each other twice: once as white and once as black. At the end of the tournament it turned out that all the players had the same number of points. Prove that there are two participants who won equal numbers of games as white.

   (1 point is awarded for a win, $\frac{1}{2}$ for a draw, 0 for a loss.)

3. Lloyd and Vladimir play a game with 100 sugar cubes placed in 50 boxes (not necessarily 2 in each box!). On his turn, each player can take any sugar cube and eat it. Lloyd goes first. Prove that Vladimir can ensure that the last two remaining cubes will be in the same box.

   (Hint: Show that Vlad can play so that after the kth move at least k boxes are empty.)