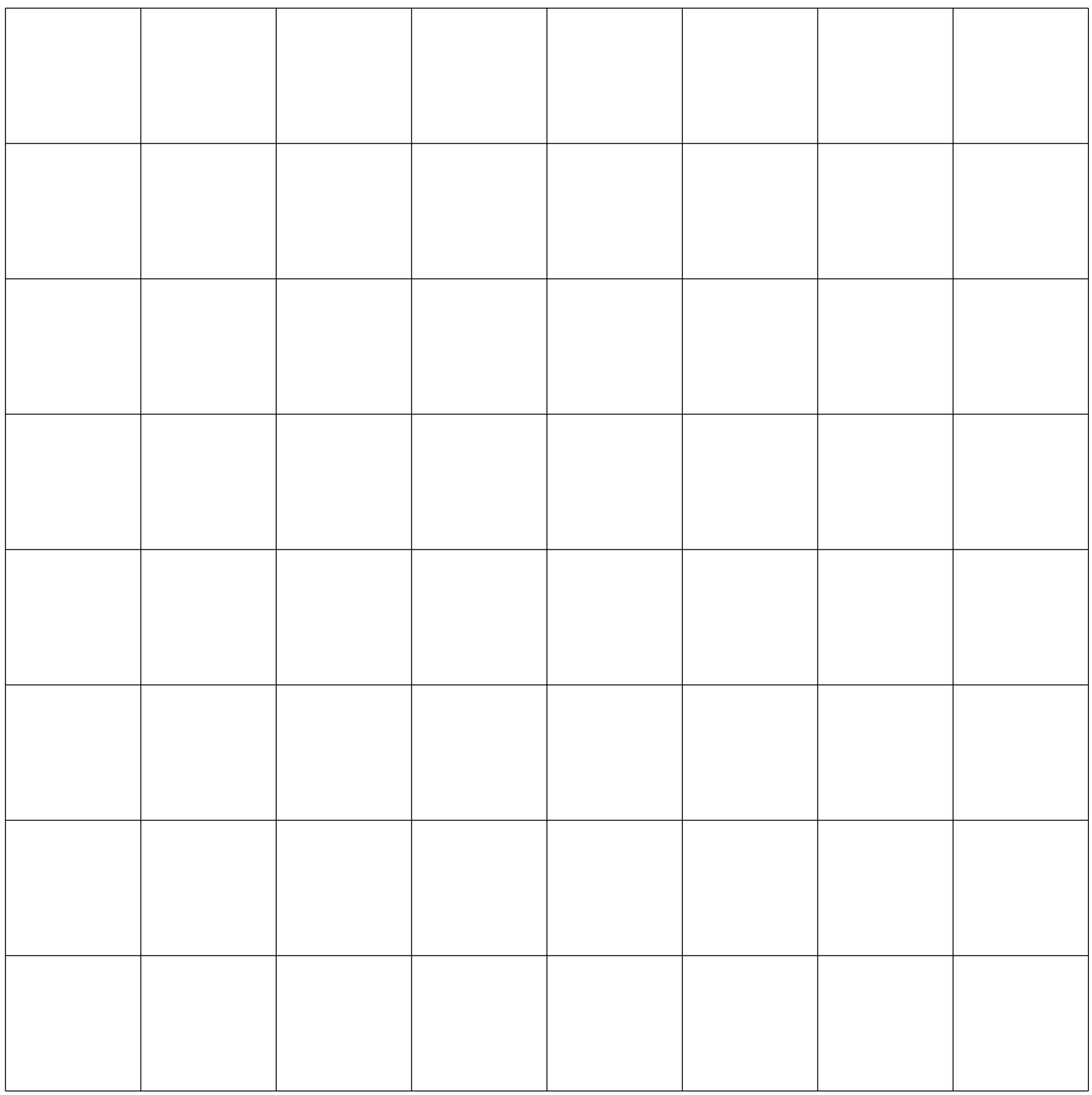
UW Math Circle

May 15, 2014

1. Find a partner and play the following game on an 8×8 chessboard. Start with a queen on square b1. On your turn, you can move the queen any number of squares vertically, horizontally, or diagonally, but she must always move up and/or to the right. The winning player is the one who can move the queen to square h8. Who wins?

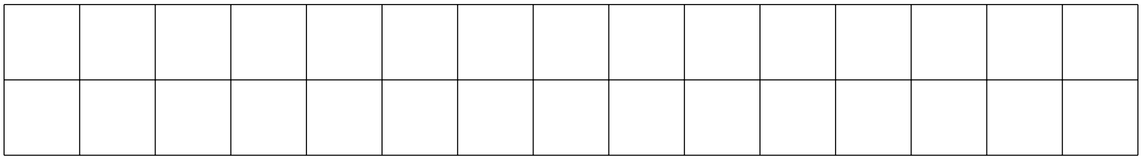


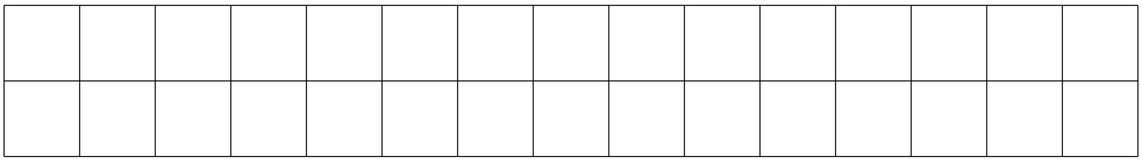
2. Ten ones and ten twos are written on the blackboard. Alice and Bob play a game in which they take turns erasing numbers from the board two at a time. If a person erases two different numbers from the board, they replace them with a 1. If they erase two of the same number from the board, they replace it with a 2. Alice wins the game if the last number remaining on the board is a 1. Bob wins if the last number left on the board is a 2. If Alice goes first, who wins the game?

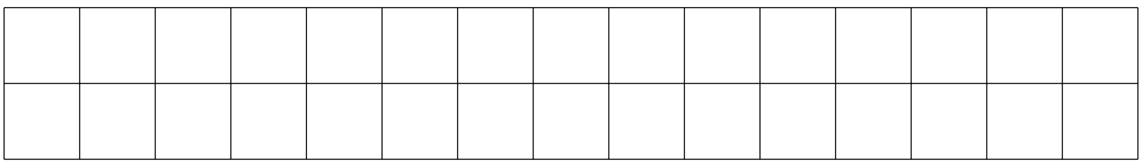
3. Find a partner and play this game. Start with three piles of stones – one with 10 stones, one with 20 stones, and one with 30 stones. On your turn, you can take as many stones as you want from one pile. The person who wins the game is the one who takes the last stone. Which player wins?

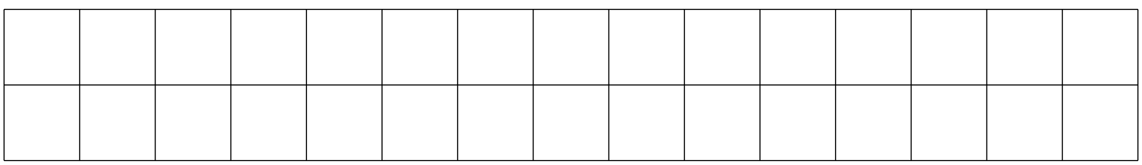
   [What happens if you change the rules so that the person who loses the game is the one who takes the last stone?]

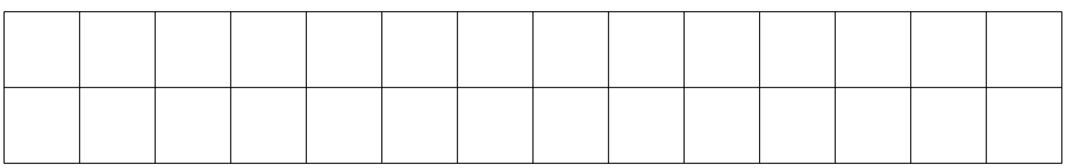
4. Gillian and Harold are playing a game on a 2×15 grid. They take turns tiling it with 1×2 dominoes, and the first person who can't place a domino loses. If Gillian goes first, who wins? What if they used a 2×14 board instead?

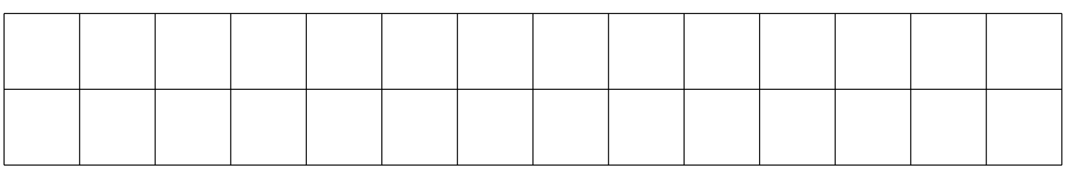


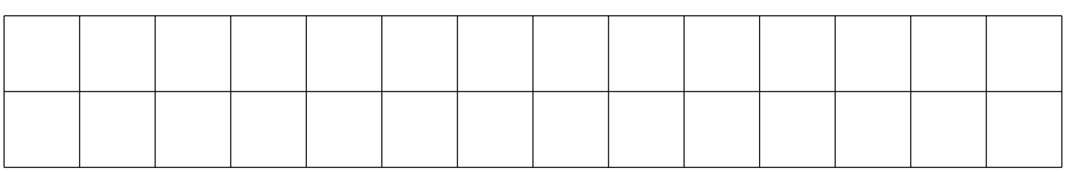












1. Is it possible to draw some number of diagonals in a convex hexagon so that every diagonal crosses EXACTLY three others in the interior of the hexagon? (Diagonals that touch at one of the corners of the hexagon don't count as crossing.)

2. Each letter in Hagrid's name represents a different digit between 0 and 9. Show that

$$HAGRID \times H \times A \times G \times R \times I \times D$$

is divisible by 3.

3. In how many different ways can you place 12 chips in the squares of a 4×4 chessboard so that

   (a) There is at most one chip in each square and

   (b) every row and every column contains exactly three chips.

4. Each cell of infinite graph paper contains one of the four numbers 1, 2, 3, or 4. Each of the four numbers is used somewhere at least once.

   Any cell has four neighbors . A cell is called *proper* if the number in that cell is equal to the number of different numbers that appear in one of its neighboring cells. In the picture on the left, the middle cell is proper because it contains the number 3 and its neighbors have three different labels (1,2, and 4). In the picture on the right, the middle cell is not proper because it contains the number 2, but its neighbors have four different labels.

	4	
2	3	2
	1	

	4	
2	2	3
	1	

   Question: Is it possible to fill the squares of an infinite piece of graph paper in such a way that every cell is proper?