UW Math Circle April 23, 2015

1. Prove that the product of any five consecutive numbers is divisible by 120.

2. Show that the fraction $\frac{2n+13}{n+7}$ cannot be reduced for any natural number n.

- 3. What is the last digit of 2014^{2015} ?
- 4. If a and b are integers that satisfy the equation 56a = 65b, prove that a + b is composite.
- 5. Can the sum of two perfect squares be another perfect square? Can the sum of three squares of positive odd numbers be a perfect square?
- 6. If p is a prime number such that $8p^2 + 1$ is also prime, what is p?



7. Challenge: If $n = \frac{a}{b}$ for integers a and b, prove that the decimal representation of n either terminates or eventually repeats.